THE EVOLUTION OF DEATH RATES AND LIFE EXPECTANCY IN DENMARK

SØREN FIIG JARNER, ESBEN MASOTTI KRYGER, AND CHRESTEN DENGSØE

ABSTRACT. From 1835 till today Denmark has experienced an increase in life expectancy at birth of about 40 years for both sexes. Over the course of the last 170 years life expectancy at birth has increased from 40 years to 80 years for women, and from 36 years to 76 years for men, and it continues to rise.

Using a new methodology we show that about half of the total historic increase can be attributed to the sharp decline in infant and young age death rates up to 1950. Life expectancy gains from 1950 till today can, on the other hand, primarily be attributed to improvements in the age-specific death rates for the age group from 50 to 80, although there is also a noticeable contribution from the further decline in infant mortality over this period. With age-specific death rates up to age 60 now at a very low absolute level substantial future life expectancy improvements must necessarily arise from improvements in age-specific death rates for ages 60 and above. Using the developed methodology we quantify the impact of further reductions in age-specific mortality.

Despite being one of countries with the highest life expectancy at the beginning of the 20th century and despite the spectacular historic increase in life expectancy since then Denmark, in fact, is lacking behind compared to many other countries, notably the other Nordic countries. The main reason being an alarming excess mortality for cause-specific death rates related to ischaemic heart diseases and, in particular, a number of cancer diseases. Age-specific death rates continue to improve in most countries, and a likely scenario is that Denmark in the future will experience improvement rates at the international level or perhaps even higher as a result of a catch-up effect.

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1. Introduction

Death is, if not defeated, then on retreat and has been so for a very long time indeed. The most striking feature of the evolution of death rates in Denmark, and many other countries, is the constant improvement over at least the last two hundred years for which we have reliable data. Moreover, there is no sign of mortality rates levelling out or improvement rates even slowing down in any near future.

The continuing mortality improvements have far-reaching consequences for pension funds and the future financing of public health care and state pension system. Being tax-paid the challenges for public financing are aggravated by low fertility rates causing the ratio between old people and young people to increase over time. In Denmark this development has not gone unnoticed and the heavy implications for the future financing of the so-called Danish welfare system are analyzed in Danish Welfare Commission (2004), which also provides recommendations for easing the future financing burden. The analysis rests, among other things, on a mortality forecast based on the Lee-Carter methodology which predicts a very modest increase in life expectancy of about 4 years for both sexes over the next 50 years, Haldrup (2004).

One of the recommendations of the Danish Welfare Commission has recently been implemented. The state pension retirement age will in the future follow the development in life expectancy. The current retirement age of 65 will be in effect till 2023 and then increase with six months each year to 67 in 2027. From then on future life expectancy gains will be reflected in a similar increase in the retirement age.

The majority of Danish pension plans consists of funded systems. These pension funds are facing two distinct problems in relation to falling mortality rates. The first problem concerns providing adequate reserves for annuity contracts already entered. In most contracts currently in effect the terms, including mortality assumptions, cannot be changed over the course of the contract. Many of these contracts are based on the, at the time, conservative technical basis G82 which has an assumed life expectancy of about 77 years for women and about 73 years for men. As reality has overtaken these assumptions a funding problem has arisen.

The second problem concerns assessing the future mortality pattern in order to base new contracts on more robust mortality assumptions. From the point of view of the pension industry this problem can to some extent be handled through securitization. There has been some academic progress in this area, see e.g. Dahl *et al.* (2007); Cairns *et al.* (2005); Lin and Cox (2005), but the market for longevity bonds and related products is still in its infancy. Another popular approach is to avoid the problem altogether by changing the products into "mortality free" savings products, or into annuities in which the mortality assumptions can be changed. However, the

basic problem of assessing the future length of the retirement period still persists.

Despite the overall picture of constantly declining mortality rates there has been periods with no improvements or even slight increases in mortality for certain age groups. One such period lasted from around 1980 to 1995 during which life expectancy rose by only about 1 year. This period with almost stagnation in life expectancy was also observed to some extent in other countries but it was more pronounced in Denmark. In fact, the slower Danish pace of improvement called for political action and in 1992 the Life Expectancy Commission was formed by the Danish Ministry of Health. In a series of reports they documented Danish excess mortality for a number of heart and cancer diseases, see e.g. Life Expectancy Commission (1998). For related work see the very comprehensive, descriptive analysis of the evolution of Danish mortality in Andreev (2002) and the discussion in Hansen *et al.* (2006).

The purpose of the present paper is to provide an overview of the evolution of age-specific death rates and to explore the link between improvements in age-specific death rates and life expectancy gains. Life expectancy (at birth) is a much used statistic used to summarize a given life table in a succinct way. Normally, this quantity is calculated for a given period life table, i.e. the mortality pattern of a population in a given calender year, and used in this way it conveys information about the general level of mortality in the population at that instant in time. It does not, however, correspond to the expected life time of a newborn, except in the hypothetical situation with no future improvements in death rates.

We develop a new sensitivity measure which relates improvements in age-specific death rates to life expectancy gains (the measure corresponds to the functional derivative of life expectancy with respect to the mortality curve). Using this tool we can quantify the historic and future contributions to life expectancy gains by the different age groups and quantify statements like "life expectancy gains used to be caused by falling infant mortality, but are now due to improvements in old-age mortality".

We will also contrast the evolution of Danish mortality with the international development and compare cause-specific mortality rates for selected countries.

2. Data and notation

Data used in the paper originate from the Human Mortality Database[1] (HMD), which offers free access to updated records on death counts and exposure data for a long list of countries. The database is maintained by University of California, Berkeley, United States and Max Planck Institute for Demographic Research, Germany.

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[1]See www.mortality.org

The data consists of gender specific death counts, $D(t, x)$, and the corresponding exposures, $E(t, x)$, for a range of years t and ages x. More precisely, $D(t, x)$ counts the number of deaths occurring in calender year t among people aged x last birthday, and $E(t, x)$ gives the total number of years lived during calender year t by people of age x. For readers familiar with the Lexis diagram, $D(t, x)$ counts the number of deaths in the square $[t, t+1) \times [x, x+1)$ of the Lexis diagram and $E(t, x)$ is the corresponding exposure.

For Denmark we are fortunate to have data from 1835 onwards making the Danish series one of the longest data series available in HMD. Recent population data for Denmark is of a very high quality being based on the Central Population Register (CPR). The register was introduced in 1968 and used for the first time in the 1976 census. Before that time censuses were held every five years with varying levels of detail in the recording of ages. The pre-1976 data is therefore based on interpolation in both the time and age dimension and also extrapolation at high ages, since the maximal age recorded has varied over time. The interested reader is referred to Andreev (2002) and Wilmoth *et al.* (2005) for a detailed account of the structure of the underlying data and the methods used to create the HMD data series.

From the death counts and exposures we form the (crude) death rates

$$(1) \qquad m(t, x) = \frac{D(t, x)}{E(t, x)} \quad \text{for } t = 1835, \dots, 2006, \ x = 0, \dots, 99,$$

which will form the basis of our analysis. Death counts and exposures are available also for ages $100, \dots, 109, 110+$, where the last age is an open-ended interval covering age 110 and above. However, as these data are very noisy and, for the early part of the series, constructed from age 100+ data we choose to ignore these also for more recent years. Hence, we let "$m(t, x) = \infty$" for $x \geq 100$ and all t, meaning that the highest attainable age is 100 years. Life expectancy at birth is only marginally influenced by this assumption even in recent years, but the life expectancy at very old ages will be slightly underestimated.

2.1. **Force of mortality.** The original data is aggregated over calender years and age groups of one year. However, it turns out to be convenient to work with a continuous formulation, in particular, when discussing the sensitivity of life expectancy to changes in mortality.

For an individual with (continuous) life time T the survival function is defined as $\bar{F}(x) = \mathsf{P}(T > x)$, i.e. the probability that the person will live longer than x years. The *force of mortality*, also called the *intensity* or the *hazard*, is defined as

$$(2) \qquad \mu(x) = -\frac{d}{dx} \log \bar{F}(x) = \frac{f(x)}{\bar{F}(x)},$$

where f denotes the density of the life time distribution. The force of mortality can be interpreted as the instantaneous death rate immediately after age x given survival to age x.

The survival function and the conditional survival functions given survival to age y can be expressed in terms of the intensity as

$$\bar{F}(x|y) = \mathsf{P}(T > y + x | T > y) = e^{-\int_y^{y+x} \mu(u)du},$$

and the expected remaining life time for an y-year-old can be expressed as

$$\bar{e}_y = \mathsf{E}(T - y | T > y) = \int_0^\infty \bar{F}(x|y)dx = \int_0^\infty e^{-\int_y^{y+x} \mu(u)du} dx.$$

Of particular interest to us is the quantity $\bar{e}_0$, life expectancy at birth.

We shall be using these formulas when calculating survival probabilities and expected life times for period life tables, $\{m(t,y)\}_{y=0,\dots,100}$, and in order to do so we define an intensity function by $\mu_t(x) = m(t,y)$ for $x \in [y, y+1)$, and base the calculations on μ_t. Thus when speaking of the life expectancy at year t we mean the quantity $\bar{e}_0(\mu_t)$, and similarly for other summary statistics related to year t.

All reported life expectancies in the paper are calculated using this procedure based on period life tables calculated from HMD data truncated at age 100. Our life expectancies might therefore deviate slightly from numbers published by the various national bureaus of statistics, e.g. Statistics Denmark.

3. Death rates

By and large death rates have been constantly improving in the historic period considered, see Figure 1. However, looking at the evolution of age-specific death rates in more detail reveals a more subtle structure with great variability in both the pace of improvements over time and the age groups affected. In the following we will give an account of the age-specific improvements and we will relate these to life expectancy gains using the decomposition technique developed in Section 4.

3.1. **Infant and child mortality.** In 1835 infant mortality was about 19% for girls and 24% for boys; thus only four out of five babies survived their first year of living. Young age mortality was also very high and life expectancy at birth at that time was only about 40 years for women and 36 years for men. During the late 19th century infant mortality declined somewhat but it still remained very high. At the turn of the century infant mortality had fallen to 13% for girls and 16% for boys. At the same time life expectancy had risen to 53 years for women and 50 years for men. However, this life expectancy gain can be contributed mainly to the decline in child mortality (between age 1 and 10) which dropped by 70% from 1835 to 1900. Figure 5 shows that the reduction in child mortality contributed with about twice as much as the reduction in infant mortality to the life expectancy gain over

the period. The reader is referred to Section 4 for a thorough description of how the age-specific life expectancy contributions in Figure 5 are calculated.

The turn of the century marked the beginning of uninterrupted improvements in infant mortality. From 1900 onwards infant mortality has been declining with about 3% each year. In 1950 infant mortality had dropped to 3% for girls and 4% for boys. During the first half of the 20th century life expectancy rose to 72 years for women and 69 years for men; about one third of the increase can be attributed to the reduction in infant mortality. The improvements in infant mortality continued after 1950 and in 2006 infant mortality had been further reduced to 0.3% for girls and 0.4% for boys. The life expectancy in 2006 was 80 years for women and 76 years for men. However, only a smaller part of the life expectancy increase after 1950 was due to reductions in infant mortality.

The historic development in infant mortality has been truly remarkable. Coming from a level in 1835 corresponding to the death rate of an 80-year-old the level has been dramatically reduced in both absolute and relative terms. In 2006 infant mortality was at the same level as the mortality of an 50-year-old. Infant mortality seems to continue to fall, however, being now at a low absolute level future reductions will not have a great impact on life expectancy.

The evolution of child mortality is very similar to the development in infant mortality with respect to the timing and size of improvement rates. This can be seen from Figure 1 where the curves for infant and 10-year-old's mortality have developed almost in parallel. However, the improvements in child mortality started already around 1870, some 30 years before infant mortality began to decrease.

Child mortality in 1835 ranged from 6% for 1-year-old boys and girls to 1% for 10-year-old boys and girls. By 1900 these death rates had been reduced to about 2% for 1-year-olds and 0.3% for 10-year-olds. About half of the life expectancy gain over this period can be attributed to this reduction. Apart from the fall in infant mortality from 1900 to 1950 the reduction in child mortality in the late 19th century is the single most important contribution to the increase in life expectancy from 1835 to 2006.

From 1900 onwards child mortality steadily declined to below 0.3% in 1950 for both boys and girls and further down to below 0.04% in 2006. The fall from 1900 to 1950 had an appreciable effect on life expectancy of about 3 years, while the impact in terms of life expectancy gains of the fall in the second half of the 20th century has been minimal. As for infant mortality child mortality is now at a very low level and further reductions will have only marginal effects on life expectancy.

In combination more than half of the 40 years gain in life expectancy since 1835 can be attributed to improvements in infant and child mortality.

3.2. **Young and middle age mortality.** The evolution of young (ages 11 to 30) and middle age (ages 31 to 60) mortality exhibits considerably variation between the sexes and across different time periods and ages. Whereas the evolution of infant and child mortality shows a fairly regular pattern with a steady decrease of the same magnitude for both sexes since the late 19th century, the improvement in young and middle age mortality is characterized by periods with high rates of improvement and periods with virtually no improvements.

Generally, the improvement rate is decreasing with age. Even for the periods with high rates of improvement in young and middle age mortality these rates are below those observed for infant and child mortality. This also holds true for old age (above age 60) mortality which has had the lowest rates of improvement of all age groups; a point to which we shall return below.

We have informally, i.e. not based on any objective measure apart from our judgement, divided the historic period into six subperiods (indicated by the vertical lines in Figure 1) in which the age-specific rates of improvement appear approximately constant. Since the death rates are plotted on a logarithmic scale this corresponds to the death rates approximately following straight lines with age-specific slopes within each subperiod.

We mention in passing that the age-specific log-linear structure within each subperiod is the assumption underlying the popular Lee-Carter model, Lee and Carter (1992). It is quite clear, however, that in case of Danish mortality estimates and forecasts based on this methodology will be very sensitive to the chosen data window as the improvement rates vary considerably over time, cf. Hansen *et al.* (2006) for a quantification of this effect.

For each age x and each subperiod the constant, annual rate of improvement has been calculated, i.e. the number r_x such that $m(t, x) = m(s, x)(1 - r_x)^{t-s}$ for the period from s to t. Within each subperiod and within each of the age groups 0, 1-10, 11-30, 31-60, 61-99 the median rate of improvement across ages was then calculated. The results are shown in Table 1. The last row contains the median rate of improvement within each age group for the whole period. We chose to use the median rather than the mean to get more robust results less sensitive to outlying rates of improvement at specific ages.

As for infant mortality the rates of reduction in young and middle age mortality were relatively modest during the 19th century. The annual rate of improvement was somewhat below 1% for most age groups. Still, from 1835 to 1900 even this low rate of improvement gave rise to mortality rates being reduced by about 50%. However, the combined impact of this reduction for the ages 11 to 60 on the gain in life expectancy over this period was only 4 years or 2 years less than the effect of the decrease in child mortality alone, cf. Figure 5. The reason the effect was not more pronounced is that the level of infant and child mortality was still very high in 1900. More than 20% died before the age of 20 and these people did not benefit from the improved mortality rates at higher ages.

Period	Females					Males				
	0	10	30	60	99	0	10	30	60	99
1835-1870	1.0	1.1	0.5	0.6	-0.1	1.0	1.3	0.7	0.8	0.1
1870-1900	0.2	2.6	1.1	0.8	-0.2	0.2	2.4	0.5	0.4	-0.2
1900-1950	3.1	4.4	4.0	1.8	0.5	2.9	3.8	3.0	1.8	0.7
1950-1980	4.1	3.3	1.4	0.7	1.5	4.5	2.9	0.2	-0.1	0.2
1980-1995	3.3	2.0	2.2	0.8	0.1	3.3	3.2	1.1	1.0	0.4
1995-2006	2.4	3.9	5.3	3.4	1.7	2.5	6.5	3.3	2.2	2.2
1835-2006	2.3	2.9	2.1	1.2	0.5	2.3	2.8	1.5	1.1	0.4

TABLE 1. Median annual rates of improvements (in percentages) for selected periods and age groups: 0, 1-10, 11-30, 31-60, 61-99.

The first half of the 20th century was a period with unprecedented mortality improvements across ages. Infant and child mortality fell sharply and young age mortality also fell rapidly at a pace of 3% or more a year, with female mortality declining slightly more than male mortality. From 1900 to 1950 mortality rates were reduced by 80% or more. Also middle age mortality fell noticeably at a rate of just below 2% a year. The effect on life expectancy over this period was an impressive increase of 18 years for women and 19 years for men; life expectancy rose from 53.4 years to 71.5 years for women and from 50.1 years to 69.1 years for men. Of this gain about 7 years can be attributed to the reduction in young and middle age mortality.

The period from 1900 to 1950 saw the largest general increase in mortality, but it also witnessed some of the greatest disasters in human history. The influenza pandemic known as the Spanish Flu in the aftermath of the World War I killed somewhere between 20 and 40 million worldwide in the years 1918-1919. The effect of the influenza was particularly devastating because of its high morbidity for the ages 20 to 40, while influenza normally is most deadly for children and elderly people. The impact of the influenza is clearly visible on Figure 1 (the cholera epidemic in 1853 can also be identified on the plot, although, much less prominent).

The other tragic event of the period was World War II. The effect of the war can be seen as spikes in the death rates for ages 20 to 40 around 1945. Both these events caused a period with high excess mortality in young age groups, but death rates quickly fell back to previous levels from where they continued to decline. Therefore neither of the events affect the mortality improvements when measuring from 1900 to 1950.

After the strong decline in mortality for both sexes up to 1950 came a period of about 30 years in which improvement rates for young and middle age mortality diverged for females and males. Female mortality continued to decline although at a slower pace, while male mortality ceased to improve.

In fact, from 1950 to 1980 male death rates slightly *increased* for ages 50 to 75.

The life expectancy gap between women and men was 3 years in 1950. A difference which had been roughly constant since 1835. As a consequence of the stagnation in male mortality improvements and the continuing female improvements this gap widened to 6 years from 1950 to 1980; over the course of the 30 years life expectancy rose from 71.5 years to 77.2 years for women and from 69.1 years to 71.2 years for men.

Like Denmark many other developed countries also experienced a deceleration in life expectancy gains from 1950 to 1980. In Denmark, however, the slow increase was followed by almost stagnation in life expectancy, particularly for women, from 1980 to 1995, while in most other countries life expectancy improvements began to pick up again around 1980. Danish women had a life expectancy improvement of only 0.6 years over these 15 years, while male life expectancy rose by 1.6 years. The life expectancy gap between women and men thereby narrowed by 1 year to about 5 years.

Looking at Figure 1 and Table 1 we see that improvements in infant, child and young age mortality in fact persisted throughout the period, but the effect on life expectancy was hardly noticeable since death rates at these ages were already very low.

From 1995 to the time of writing Denmark has again experienced high rates of improvements, and for the first time in history death rates are improving simultaneously for all age groups including the oldest ages. The improvements, particularly in middle and old age mortality, have caused a life expectancy increase of 2.7 years for women and 3.2 years for men. Over the last decade the life expectancy gap between women and men has thus been further reduced by half a year. The life expectancy in 2006 was 80.5 years for women and 75.9 for men.

The current rate of improvement in death rates and life expectancy is historically high, only exceeded by the improvements observed from 1900 to 1950. Since 1995 there has been an average, annual increase in life expectancy of 0.24 years for women and 0.29 years for men. In comparison, there has been an average, annual increase over the whole period of about 0.23 years for both sexes. This average, of course, includes the spectacular period from 1900 to 1950 during which there was an average, annual increase of about 0.37 years for both sexes. Thus, apart from this period the average, annual increase has been considerable lower and in this perspective the current level of improvement is indeed very high.

3.3. **Old age mortality.** As mortality rates decline and life times increase the perception of "old age" also gradually changes. In 1835 with less than 40% of newborns reaching the age of 60, cf. Figure 2, this was certainly considered a very old age. Nowadays, with about 90% reaching age 60 a person of this age is no longer "old", but merely an adult. However, for the

sake of this paper we take old age to mean ages 61 and above as a compromise reflecting the historic period as a whole.

Through the 19th century death rates for 70-year-olds did not improve, while for even older age groups they actually increased. We should, of course, treat this finding with caution as we must consider data from this early period less reliable and to some extent prone to age misreporting, in particular, at high ages.

During the first half of the 20th century old age mortality started to improve but at a rate of less than 1% a year for males and even lower for females. These rates were much lower than those observed for the younger ages. Of the historic increase in life expectancy of about 19 years from 1900 to 1950, less than 2 years can be attributed to improvements in old age mortality, cf. Figure 5.

The pattern of no improvement in male mortality and continued improvement in female mortality seen from 1950 to 1980 at ages below 60 is also seen in old age mortality to an even wider degree. In this period female death rates for age 70 and 80 declined with almost 2% a year and with 1% for age 90. At the same time, male death rates for the old ages decreased only marginally or, in the case, of the 70-year-olds increased slightly.

As for the younger ages the period from 1980 to 1995 saw hardly any improvements in old age mortality. However, since 1995 old age death rates have been steadily declining and the current rate of about 2% a year is the highest in history. Of the life expectancy gain of about 13 years for both sexes from 1950 to 2006 reductions in old age mortality has contributed with about 4.5 years for women and about 2.5 years for men.

From the start of the data period till today old age mortality has in general improved at a slower pace than the younger age groups and the improvements have occurred later in time. Also the effect on life expectancy has been moderate compared to, in particular, the reductions in infant and child mortality. However, with death rates now at quite low levels up to age 60 future life expectancy gains will come almost exclusively from improvements in old age mortality.

3.4. **Life time distributions.** Another perspective on the aggregated effect of the evolution of death rates can be obtained from looking at the life time distributions implied by period life tables. As for life expectancy calculations life time distributions based on period life tables do not represent the actual distribution of the life length of newborns (unless no further improvements occur). Still the distributions give a useful snapshot of the current mortality pattern of the population.

Figure 2 depicts life time distributions at selected years corresponding to the endpoints of the subperiods identified above. These years represent structural breaks in (close to) constant improvement regimes. For each year the plot shows the probability of dying before the age of 20, 40, 60, 70, 80,

90 and 100, i.e. the cumulative distribution function of life length evaluated at these ages.

The combined effect of declining death rates for infants and children is clearly visible. In 1835 the probability of dying before the age of 20 was about 40%. Over the next 115 years this probability decreased rapidly to below 2% in 1950, and further down to less than 1% in 2006. In fact, in 2006 the probability for females to die before age 40 was less than 1.5%, and about 2.5% for males.

Over the period all age-specific death rates have improved. However, the rate of improvement has not been uniform across ages. Generally, the younger ages have had the largest rates of improvements, the older ages the lowest, and the oldest-old almost no improvements at all. The effect can be seen on Figure 1 where the death rates are much more spread out in 2006 than they were in 1835.

The life time distributions provide information about the aggregate effect of improvements in age-specific death rates up to a given age. For instance, the probability of reaching age 80 in 1835 was only about 10% for women and 7% for men[2]. In 2006 these numbers had increased to 60% and 46%, respectively. The probability of reaching the age of 90 has also increased considerably over the period although far less, while the probability of becoming a centenarian has only improved from 0.2% to 1.6% for women and from 0.03% to 0.4% for men.

Historically male death rates have been about 10% higher than female death rates (plots not shown). In the period from 1950 to 1980 where male death rates stagnated while female death rates continued to decline this gap widened to an excess male mortality of about 70%. Currently, male death rates are about 50% higher than female death rates for most age groups, with the exception of young male mortality (ages 20 to 40) which is more than twice as high as female mortality. This gender difference give rise to substantially lower probabilities for males to attain high ages as seen on Figure 2.

Improvements in age-specific death rates have caused people to die at still higher ages. However, the probability of attaining a very high age of 100, say, has not improved by much. This phenomenon is sometimes referred to as rectangularization. The term refers to period survival functions looking increasingly "rectangular" staying close to 1 up to high ages and then dropping to 0 over a short age span at very high ages. In 2006 about 60% of female deaths occurred between ages 80 and 100, a span of only 20 years, while in 1835 the same percentage of deaths was spread out between ages 25 and 100.

The nature of oldest-old mortality and how it will develop in the future is the object of an interesting, but somewhat speculative, debate. Some argue

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[2]Figure 2 gives the probability of *dying* before a given age and the numbers are thus obtained as 100% minus the values in the figure.

that there is a biological highest age for the human body. This would imply that medical and other advances can improve death rates only up to a certain age and people will tend to die in a still more narrow age span just below the highest possible age. Others argue that no such upper limit exists and that mortality of the oldest-old will indeed improve in the future and still higher ages will be attained. The interested reader is referred to Thatcher (1999); Rose and Mueller (2000); Yashin and Iachine (1997) and references therein.

One should keep in mind that the development of oldest-old mortality is largely of academic interest. The practical and economic implications of even a drastic improvement among the oldest-old will be limited as this group is quite small and will continue to be so for a long time.

4. LIFE EXPECTANCY

Life expectancy at birth, or simply life expectancy, is the usual way of summarizing the age-specific death rates of a population at a given point in time. It is also the measure of choice when describing the effect of improvements in death rates over time and when comparing "the state of mortality" in different countries; in the present paper we make use of it for both purposes.

We stress again that despite the intuitive appeal of the name, the life expectancy calculated from a period life table does not represent the expected life time of a newborn. The latter quantity, the so-called cohort life expectancy, is generally, substantially higher since newborns will typically experience age-specific death rates lower than the current level due to future improvements. However, for a cohort still alive the calculation of its life expectancy must necessarily be partly subjective and based on a specific model for the as yet unknown future death rates; only for extinct cohorts can life expectancies be calculated from observed death rates only.

The main advantage of period life expectancies over cohort life expectancies is that they are objective summaries of observed death rates and for this reason we focus on the former in the present descriptive study, although, cohort life expectancies are arguably of more interest in some situations.

The life expectancy at birth depends on all age-specific death rates from age 0 to the highest attainable age which, in this paper, is set to age 100. Similarly, one can calculate the remaining life time given survival to a given age. These quantities depend on the age-specific death rates from the conditioning age onwards and they provide information about the tail of the life time distribution.

Figure 3 shows the total expected life time for females and males given survival to a given age for selected period life tables. The height of the first box indicates the life expectancy at birth, the combined height of the first and second box indicates the expected *total* life time given survival to age 20, and so on for ages 40, 60, 70, 80 and 90. Thus the height of the second box

represents the increase in total life expectancy when surviving from age 0 to age 20, the height of the third box represents the increase when surviving from age 20 to age 40, and so on for the higher ages. Note that the expected *remaining* life time at a given age can be obtained from the graph as the expected total life time minus the conditioning age.

Two features of Figure 3 stand out. The first is the sharp increase in life expectancy at birth from 1835 to 1950 (and the more moderate increase hereafter). The second is the absence of substantial improvements in expected total life time given survival to high ages.

As already noted the increase in life expectancy at birth from 1835 to 1950 was primarily due to a marked reduction in infant and child mortality over the period. The very high level of infant and child mortality in the 19th century can indirectly be seen on Figure 3. In 1835 male life expectancy at birth was 36 years, while the expected total life length given survival to age 20 was 59 years. Thus surviving the first 20 years gave you an expected gain in total life time of 23 years! A discrepancy of that size implies that the chance of surviving to age 20 must have been rather small. In fact, as can be seen from Figure 2 about 40% of newborn males died before age 20. Hence the high level of infant and child mortality manifests itself as large total life expectancy gains from surviving the first 20 years, i.e. in the large size of the second box.

After 1950 we see a very different pattern in which life expectancy at birth is almost the same as the expected total life length given survival to age 20 and 40. The additional expected total life time given survival to age 60 and 70 is also quite small. The new pattern that has arisen is caused by all age-specific death rates up to age 40, say, now being at a very low level. The almost collapse of expected total life time given survival up to age 60 means that future gains in life expectancy at birth will be mirrored almost one to one in gains in expected life time of 60-year-olds.

The second striking feature of Figure 3 is that the remaining expected life time for 90-year-olds has been almost constant at a level of about 4 years throughout the period (of course the picture is slightly exaggerated by our setting the death rate at age 100 to 1). Gains in expected remaining life time for 80-year-olds have also been modest. It has increased from 5 years in 1835 to 7 in 2006 for males and from 6 years to 9 years for females. Even the expected remaining life time for 60-year-olds has increased with only 7 years for males and 8 years females over the period from 1835 to 2006. However, the increase since 1995 has been about 2 years. Overall, the aggregate effect of improvements in old age mortality have been fairly modest and the gains in terms of expected remaining life time have occurred quite recently.

The purpose of the rest of this section on life expectancy is to develop a sensitivity measure which relates improvements in age-specific death rates to increases in life expectancy and to use this to decompose historic life expectancy gains into age-specific contributions. A tool which we have already used in Section 3.

4.1. **Sensitivity measure.** Life expectancy is a complicated function of the entire intensity curve and it is not easy a priori to say how it will respond to changes in (part of) the curve. Apart from infant and child mortality the force of mortality is increasing with age and reaches very high levels at old ages. This, however, does not imply that improvements in old age mortality will have an appreciable effect on life expectancy since only few people will benefit from the improvements.

For any age x we can express life expectancy at birth as a term related to those dying before age x and a term related to those surviving to age x. Using the notation introduced in Section 2 we have

$$\bar{e}_0 = \mathsf{E}(T|T \le x)\,\mathsf{P}(T \le x) + \mathsf{E}(T|T > x)\,\mathsf{P}(T > x).$$

Changing the intensity for ages higher than x will affect the expected total life time given survival to age x, $\mathsf{E}(T|T > x)$, but the effect on life expectancy at birth, $\bar{e}_0$, will be dampened by the probability of surviving to age x, $\mathsf{P}(T > x)$.

Note that the probability of surviving to, or dying before, a given age depends only on the part of the intensity curve *before* that age, while the expected remaining life time, or the expected total life time, given survival to a given age depends only on the part of the intensity curve *after* that age. In that sense the life time distribution and the expected total life times, Figures 2 and 3, are dual representations of the same information.

The formula above is valid for one age at a time and can be used to derive a sensitivity measure for changes above a given age. However, to understand the simultaneous impact of changes to the entire curve we will form the (functional) derivative which measures the rate with which life expectancy will change when changing the intensity curve in a given direction. We will consider age-specific *relative*, *improvements* of rate δ of the intensity curve μ and thus calculate the following derivative

$$
\begin{aligned}
\frac{\partial \bar{e}_0\left(\mu(1-\epsilon\delta)\right)}{\partial \epsilon}\bigg|_{\epsilon=0} &= \frac{\partial}{\partial \epsilon}\int_0^\infty e^{-\int_0^x \mu(y)(1-\epsilon\delta(y))dy}dx_{|\epsilon=0} \\
&= \int_0^\infty \int_0^x \mu(u)\delta(u)e^{-\int_0^x \mu(y)dy}dudx \\
&= \int_0^\infty \int_u^\infty \mu(u)\delta(u)e^{-\int_0^x \mu(y)dy}dxdu \\
&= \int_0^\infty \delta(u)D_\mu(u)du,
\end{aligned}
$$

where the kernel is given by

$$(3) \qquad D_\mu(u) = \mu(u)\int_u^\infty e^{-\int_0^x \mu(y)dy}dx = \mu(u)\bar{F}(u)\bar{e}_u = f(u)\bar{e}_u.$$

The kernel measures the (marginal) effect of a relative improvement of the intensity at a given age, u, and is equal to the fraction of people dying at that age, measured by the density $f(u)$, times the expected remaining life

time $\bar{e}_u$. The result is very intuitive but could hardly have been anticipated in advance. Note that since working in a continuous framework one has to integrate over all the age-specific improvements using the kernel as a weight function to get the aggregate effect, and that improvements at one age only has no effect.

For all years from 1835 to 2006 we have calculated the kernel using formula (3) and subsequently computed the average over the periods 1835–1899, 1900–1949 and 1950–2006. The result is shown in Figure 4. In the 19th century the sensitivity of life expectancy to improvements in age-specific mortality was almost monotone decreasing in age, apart from the hump for young males. This was to be expected due to the high level of infant and child mortality and the low fraction of people reaching high ages.

In the first half of the 20th century with infant and child mortality much reduced the highest sensitivity is now to be found for ages 50 to 80, although life expectancy gains from improvements in young age mortality is still substantial. The kernel has a value of about 0.13 for females between age 20 and 40, which means that a simultaneous improvement of 10%, say, of the intensity for this age group would increase life expectancy at birth with approximately 0.26 (= 0.13 · 10% · 20) years. The same reduction for the age group from 60 to 80 would have an effect almost twice as high.

Apart from the peak at age 0 the sensitivity curve for the last period is shifted further towards the high ages and has a well-defined hump around age 75 for women and age 70 for men. As opposed to the previous periods improvements in the mortality at very high ages will now have an appreciable effect on life expectancy at birth. For females the sensitivity at age 85 is about the same as at age 50 and the same relative improvement in the two age-specific death rates will therefore have the same impact on life expectancy at birth.

4.2. **Decomposing life expectancy gains.** The decline in age-specific mortality rates over time causes life expectancy to rise and it is illuminating to study how the different age groups have contributed to the increase. Below we suggest a way to decompose life expectancy gains, a tool which we have already used repeatedly throughout Section 3.

Generally all age-specific death rates will be different when comparing two life tables and a first attempt of decomposing the life expectancy difference could be to decide on an order, e.g. from the lowest age to the highest age, in which to change the death rates from one table to the other and assign the change in life expectancy after each change to that age. The result of this procedure, however, will depend on the chosen order and one would arrive at a different result when changing the updating scheme.

Having access to only two period life tables at time s and time t, say, there is no unique way to obtain the desired decomposition. However, imagine the idealized situation in which we could observe each of the age-specific intensities at any time u between s and t, $\mu_u(y)$, and assume that the transition

from $\mu_s(y)$ to $\mu_t(y)$ is smooth. Under these assumption we can calculate the time derivative of life expectancy using a similar line of reasoning as in the previous section

$$
\begin{aligned}
\frac{\partial}{\partial u}\bar{e}_0(\mu_u) &= \int_0^\infty \frac{\partial}{\partial u} e^{-\int_0^x \mu_u(y)dy} dx \\
&= -\int_0^\infty \int_0^x \mu_u'(v) e^{-\int_0^x \mu_u(y)dy} dv dx \\
&= -\int_0^\infty \int_v^\infty \mu_u'(v) e^{-\int_0^x \mu_u(y)dy} dx dv \\
&= -\int_0^\infty \frac{\mu_u'(v)}{\mu_u(v)} D_{\mu_u}(v) dv,
\end{aligned}
$$

where the kernel, $D_{\mu_u}(v)$, is defined in (3). The change in life expectancy from time s to time t, $\bar{e}_0(\mu_t) - \bar{e}_0(\mu_s)$, can then be written as

$$
\begin{aligned}
\bar{e}_0(\mu_t) - \bar{e}_0(\mu_s) &= \int_s^t \frac{\partial}{\partial u}\bar{e}_0(\mu_u) du \\
&= -\int_s^t \int_0^\infty \frac{\mu_u'(v)}{\mu_u(v)} D_{\mu_u}(v) dv du \\
&= -\int_0^\infty \int_s^t \frac{\partial}{\partial u}\left\{\log \mu_u(v)\right\} D_{\mu_u}(v) dv du.
\end{aligned}
$$

The inner integral in the last formula can be interpreted as (the density of) the age-specific contribution to the life expectancy gain over the period related to age u, which is precisely what we are after. Note, however, that to calculate this expression we would have to make an assumption about the value of $\mu_u(v)$ for non-integer values of u, e.g. linear or exponential between the observed values at the neighbouring integers.

In order to obtain a simpler formula we will instead make the assumption that the kernel, $D_{\mu_u}(v)$, exhibits only a weak dependence on time, i.e. $D_{\mu_u}(v) \approx \bar{D}(v)$, for some $\bar{D}(v)$. Under this assumption we get the relation

$$
(4) \qquad \bar{e}_0(\mu_t) - \bar{e}_0(\mu_s) \approx \int_0^\infty \bar{D}(v) \log \frac{\mu_s(v)}{\mu_t(v)} dv,
$$

which expresses the increase in life expectancy in terms of improvements in the age-specific log mortality rates and the previously introduced kernel. Note that only the intensities at time s and t are needed under this assumption.

In Figure 5 we have used formula (4) to decompose the life expectancy gains for each of the periods 1835–1900, 1900–1950 and 1950–2006 into contributions from different age groups. For each of three periods we have used the average kernel over the period, shown in Figure 4, as $\bar{D}$ and for each age v between 0 and 100 we have then calculated its contribution to the life expectancy gain as $\bar{D}(v) \log[\mu_s(v)/\mu_t(v)]$. To make the age-specific contributions sum to the observed life expectancy gain over the period we have scaled

them by a common factor. Finally, we have grouped the contributions into the age groups 0, 1–10, 11-20, . . . , 91–100, and stacked the contributions for each period on top of each other.

The plot shows three distinct improvement patterns. In the first period life expectancy gains were driven mainly by reductions in child, and to some extent infant, mortality. Over the second period a wide range of age groups contributed to the increase in life expectancy, with the reduction in infant mortality being the single most important. While in the most recent period life expectancy gains have come mainly from improvements in high age mortality but also to some degree from further reductions in infant mortality.

5. Denmark and the World

Mortality improvements is by no means an isolated Danish phenomenon. The evolution of Danish mortality which has resulted in an increase of 40 years in life expectancy from 1835 to 2006 is certainly remarkable, but many developed countries have seen even larger improvements over that period. In fact, the evolution in large parts of the developed world has surpassed the Danish evolution to such an extent that over the course of the last century Denmark has moved from being a top-ranking country with respect to life expectancy to currently being in the bottom half. This development has caused concern not least because other Nordic countries, e.g. Sweden and Norway, have been able to maintain their position as world-leading countries with respect to life expectancy.

All countries have had their own unique mortality history depending on national characteristics. Therefore, to put the Danish development into an international perspective without going into country-specific details we have constructed a single, international death rate based on data for 18 developed countries[3].

Data is extracted from the Human Mortality Database and for each country it consists of gender specific death counts and exposures in the format described in Section 2. From these we have constructed an international death rate as the ratio between the total death count and the total exposure in all of the countries for which data exists for the given year. To ensure that most of the larger countries are represented each year we consider only the time period from 1900 to 2004.

The plots in Figure 6 compare Danish and international death rates for adult ages, and Tables 2–3 show life expectancies for selected countries and years. At the start of the 20th century Danish death rates were about 15% lower than the international level for both females and males. The Danish

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[3]The 18 countries with the time range of available data in parenthesis are: Australia (1921–2004), Austria (1947–2005), Belgium (1841–2005), Canada (1921–2004), UK, Civilian Population (1841–2003), Finland (1878–2005), France, Civilian Population (1899–2005), West Germany (1956–2004), Iceland (1838–2005), Italy (1872–2003), Japan (1947–2005), Netherlands (1850–2004), Norway (1846–2006), Portugal (1940–2005), Spain (1908–2005), Sweden (1751–2006), Switzerland (1876–2005), US (1933–2004).

life expectancy was comparable to the life expectancy in Sweden and Norway, and substantially higher than in France and UK.

Country	1900	1950	1980	1995	2004
Denmark	53.4 (3)	71.5 (3)	77.2 (6)	77.8 (7)	79.8 (7)
Sweden	53.6 (2)	72.4 (2)	78.8 (2)	80.8 (4)	82.6 (3)
Norway	55.1 (1)	73.2 (1)	79.1 (1)	81.4 (3)	82.3 (4)
France	46.7 (5)	69.2 (6)	78.4 (4)	81.9 (2)	83.8 (2)
UK	48.1 (4)	71.3 (4)	76.7 (7)	79.4 (5)	80.7 (5)
US		71.0 (5)	77.4 (5)	79.1 (6)	80.2 (6)
Japan		60.9 (7)	78.7 (3)	82.8 (1)	85.4 (1)

TABLE 2. Female life expectancy at birth for selected countries and years (rank in parenthesis). For UK last year available is 2003.

Country	1900	1950	1980	1995	2004
Denmark	50.1 (3)	69.1 (3)	71.2 (4)	72.7 (6)	75.1 (7)
Sweden	50.7 (2)	69.8 (2)	72.8 (2)	74.8 (3)	78.3 (2)
Norway	51.7 (1)	69.9 (1)	72.3 (3)	76.2 (2)	77.5 (3)
France	43.0 (5)	63.4 (6)	70.2 (6)	73.8 (5)	76.7 (4)
UK	44.1 (4)	66.5 (4)	70.7 (5)	74.2 (4)	76.4 (5)
US		65.4 (5)	70.0 (7)	72.7 (7)	75.2 (6)
Japan		57.5 (7)	73.4 (1)	76.5 (1)	78.7 (1)

TABLE 3. Male life expectancy at birth for selected countries and years (rank in parenthesis). For UK last year available is 2003.

In the first half of the 20th century mortality rates declined substantially in many developed countries. The Danish life expectancy rose by almost 19 years for both females and males and Denmark was, at that time, still one of the countries with highest life expectancy in the world. However, life expectancy rose by even more in France and UK so although Denmark remained its relative position the gap had already narrowed, particulary for women.

From 1950 to 1980 Danish female mortality continued to improve although at a lower pace than in the previous period. However, improvements in international death rates did not slow down as much and as a result Denmark was no longer a leading country with respect to life expectancy at the end of the period. Of the seven countries listed in Table 2 only UK women had a lower life expectancy than Danish women in 1980.

In the same period Danish males experienced an almost stagnation in death rates and life expectancy. International death rates, on the other hand, continued to decline and at the end of period they had essentially caught up with the Danish level. Danish male life expectancy in 1980 was still in the high end, but the gap to the other countries had shrunk substantially.

When discussing life expectancy evolution Japan stands out. In 1950 the Japanese life expectancy trailed many other developed countries by almost 10 years for both males and females. However, over a period of only 30 years they came to have one of the highest life expectancies in the world. From 1950 to 1980 life expectancy rose with 16 years for men and 18 years for women corresponding to an annual increase of over half a year. In comparison, the Danish life expectancy increased with only 2 years for men and 6 years for women.

The following 15 years from 1980 to 1995 saw a stagnation in Danish female mortality. The international level continued to fall over the period and a large gap between the Danish and the international level was established. At the end of the period the excess mortality in Denmark for women aged 50 to 70 was more than 50% compared to the international level. In 1995 Danish female life expectancy was the lowest of the seven countries in Table 2 more than one year behind US as the second lowest.

Danish males was also overtaken by the international development from 1980 to 1995, but the excess Danish male mortality at the end of the period was far less than for females. Still, in 1995 Danish male life expectancy was at almost the same level as that of US and together they ranked lowest of the seven countries in Table 3.

In Japan life expectancy continued to rise at a high rate from 1980 to 1995, and at the end of the period they ranked number one for both females and males. Hence, it took Japan less than half a century to move from the bottom of the list to the top. Truly a remarkable achievement. During the same period Denmark fell from the top to the bottom.

Since 1995 Denmark has again experienced substantial life expectancy gains. From 1995 to 2004 life expectancy rose with about 2 years for women and about 2 and a half years for men. However, most other developed countries had similar gains and the ranking of the seven countries in Tables 2–3 in 1995 and in 2004 is therefore almost identical.

In 2004 Denmark was still the lowest ranking country, while Japan, France, Sweden and Norway constituted the top half. The gap between Denmark and the two other Nordic countries was between 2 and 3 years. Compared to Japan, however, the life expectancy gap was about 2 and a half years for males and about 5 and a half years for females.

Throughout the period life expectancy gains for women have been largest in Japan. Japanese women were already the longest living in 1995 but even so Japan experienced the largest increase from 1995 to 2004 among the seven countries in Table 2. This is fascinating in its own right but it also shows that large increases can occur in countries where life expectancy is already high.

Hence, further, substantial increases in life expectancy can be expected for both Sweden and Norway. In Denmark we can hope for a future reduction in excess mortality, particularly for women, with the effect of even higher future increases in life expectancy as Denmark catches up.

5.1. **The big why.** There is no agreed upon explanation as to why the mortality progress in Denmark from 1950 to 1995 was so much slower than in most other developed countries. It is a fact, though, that this period which had the largest economic progress in history and during which the so-called Welfare State was founded with a large public health sector saw relatively modest mortality improvements.

Many explanations have been put forward trying to explain this paradox including increased female labour market participation rates in the 1960s; increased consumption of tobacco and alcohol; changes in diet and other life style changes; lack of physical exercise and increased obesity; and less efficient screening programs and treatments, particularly for cancer. None of these explanations is entirely satisfying on its own, however, since most developed countries have had a development similar to Denmark in many of these areas.

Some light can be shed on the Danish excess mortality by looking at cause-specific mortality rates. For the seven countries previously compared Table 4 shows age-standardised death rates for all causes, cancer and circulatory diseases. The latter two being the major death causes in the developed world. The reported death rates are computed as a weighted average of age-specific death rates using (the age composition of) the same standard population as weights. This is done to take account of differences in age structure of the populations being compared. Data is extracted from OECD Health Data 2007 which contains statistics on health and health care systems in OECD countries from 1960 to 2006[4]. However, for reasons of comparison we have chosen to use data from 2001 as this is the latest year for which death rates for all of the selected countries are available (the restricting country being Denmark!).

The excess mortality of Danish women is indeed alarming and much of it can be attributed to excess mortality related to cancer (malignant neo-plasms). Compared to Sweden and Norway the Danish women have a cancer related excess mortality of more than 30%, while the level of mortality related to heart diseases (circulatory diseases) is comparable in the three countries. Even when comparing with UK, which has the second highest cancer related death rate, the Danish women have an excess mortality of 20%. Once again the Japanese (and the French) women stand out by having cause-specific death rates much lower than the women in any other country.

The picture for males is less clear. First of all, the variation between the seven populations is smaller for men than for women. For both males and females Japan has the lowest all-cause death rate and Denmark the highest.

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[4]See www.sourceoecd.org

Country	Females			Males		
	All causes	Malignant neoplasms	Circulatory diseases	All causes	Malignant neoplasms	Circulatory diseases
Denmark	593 (7)	186 (7)	184 (5)	871 (7)	245 (6)	308 (6)
Sweden	460 (3)	134 (3)	182 (4)	705 (2)	180 (1)	303 (5)
Norway	481 (4)	140 (4)	171 (3)	783 (4)	208 (3)	299 (4)
France	411 (2)	116 (2)	116 (2)	774 (3)	252 (7)	203 (2)
UK	536 (5)	155 (6)	194 (6)	804 (5)	225 (5)	314 (7)
US	554 (6)	142 (5)	198 (7)	826 (6)	207 (2)	297 (3)
Japan	329 (1)	102 (1)	103 (1)	638 (1)	215 (4)	174 (1)

TABLE 4. Standardised death rates per 100,000 population based on data for 2001 (rank in parenthesis).

However, whereas Danish women have an all-cause excess mortality of 80% compared with Japan the Danish men have an all-cause excess mortality of "only" 37%. The two cause-specific death rates for Danish males are both high, but neither of them stands out. For cancer there is an excess mortality compared to Sweden and Norway, but the Danish level is comparable to that of France and UK. For heart diseases, of which ischaemic heart diseases count the most (numbers not shown), the countries can be divided in two groups: Japan and France in the top and the five other countries at a comparable level at the bottom.

Looking at the pattern of cause-specific death rates across countries it seems reasonable to conclude that diet and smoking habits must be at least part of the explanation of the observed differences between countries. Undoubtedly, numerous other factors are also important and the question of why the Danish excess mortality is so high must still be considered largely unresolved.

6. CONCLUDING REMARKS

The story of mortality evolution is one of continued improvements. Whether age-specific death rates can decline indefinitely and life expectancy continue to rise or whether there exists an unsurmountable biological barrier for human life spans is a question of philosophical nature. However, it seems almost certain that we will witness appreciable mortality improvements in the foreseeable future, in particular for countries like Denmark which is lacking behind other developed countries. Here a reduction in excess mortality will in itself give rise to substantial life expectancy gains.

Going back in history one can point at a number of factors behind the observed reductions in death rates: improvements in nutrition and sanitary

conditions, higher standards of living, better housing and working conditions, public health measures, better hygiene in hospitals, medical advances etc. Over the course of history these changes have led to death causes changing from infectious diseases such as tuberculosis, diphtheria and cholera to degenerative diseases such as cancer and heart diseases.

Detailed knowledge of causality is valuable for understanding the past but it is of limited value when trying to predict the future. The mechanism governing death rates is too complex and the impact of future medical inventions, economic development, demographic changes etc. cannot possibly be foreseen. Consequently, most mortality projections are based on purely statistical models extrapolating past trends.

The most widely used model for mortality projections is still the one proposed by Lee and Carter (1992) although numerous extensions and other model types have been proposed since then, see e.g. Brouhns *et al.* (2002); Lee and Miller (2001); Renshaw and Haberman (2006); de Jong and Tickle (2006); Currie *et al.* (2004); Cairns *et al.* (2006). For recent comparisons of selected models see Cairns *et al.* (2007) and Booth *et al.* (2006).

All these models provide more or less structured projections of age-specific death rates and their main strength is their ability to extrapolate regular improvement patterns. Changes in improvement rates are considered as structural breaks and data before the last structural break is often disregarded. Large populations, like the US, do indeed show regular patterns with near constant annual rates of improvement over long periods and this approach, although hardly optimal, is feasible. However, for small regions, like Denmark, the mortality evolution has been much more erratic with many periods with very different improvement patterns and basing a possibly long-term projection on the last regular period is neither robust nor trustworthy. Despite the wealth of models in existence we feel there is a need for developing a methodology which can make convincing projections from volatile mortality rates making proper use of all available data. In a forthcoming paper we propose a new model for small region mortality projections.
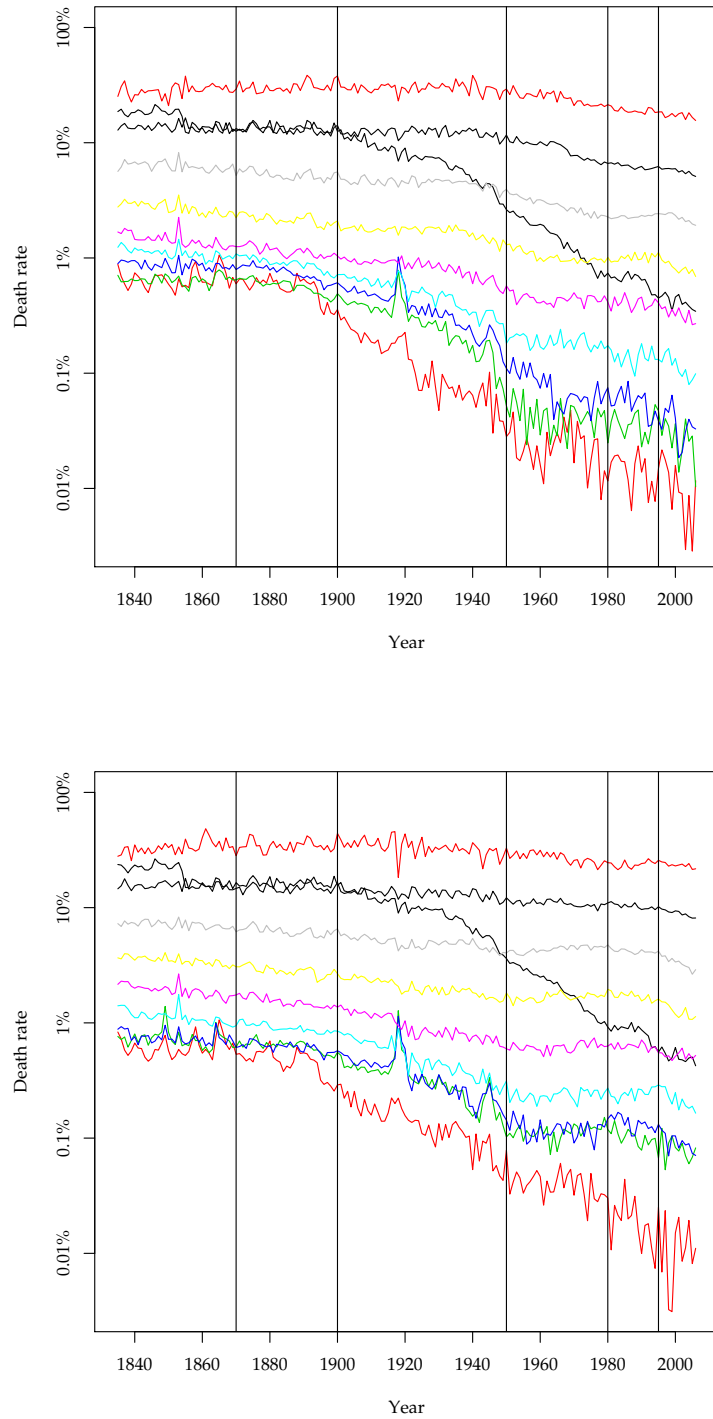
7. FIGURES

FIGURE 1. Development in Danish female (top) and male (bottom) age-specific death rates from 1835 till 2006 for ages $0, 10, \ldots, 90$. The line starting out as the second highest and crossing several others is the death rate for 0-year-olds. The other lines represent ages 10 to 90 in increasing order.
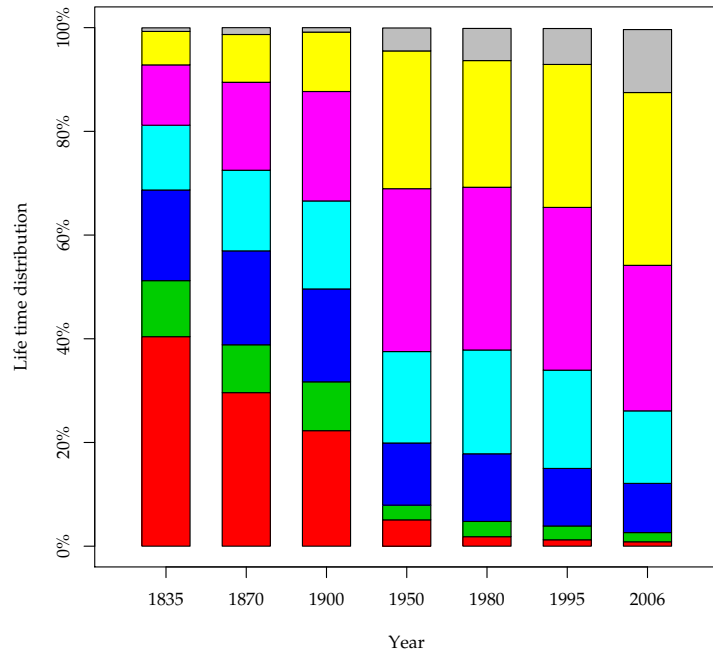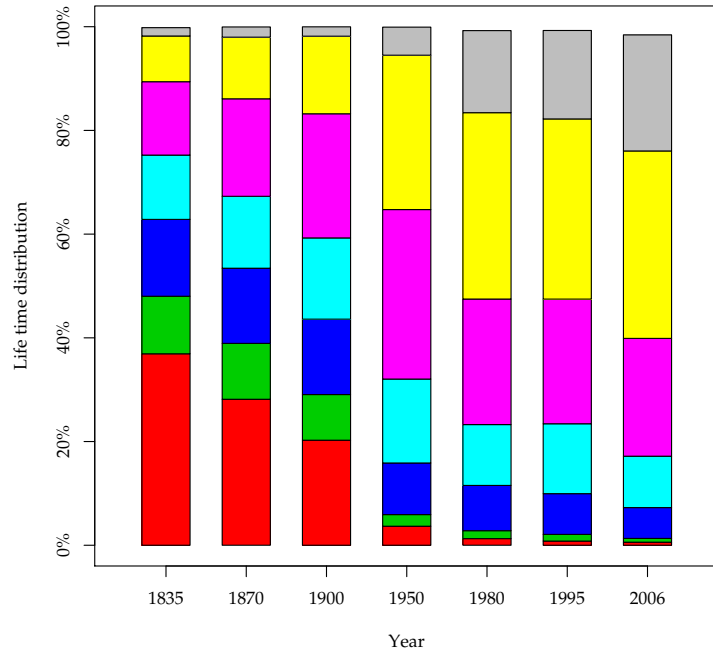
FIGURE 2. Life time distribution of Danish females (top) and males (bottom) at selected years. Each column shows the probability of dying before the age of 20, 40, 60, 70, 80, 90 and 100 based on the period life table for that year.
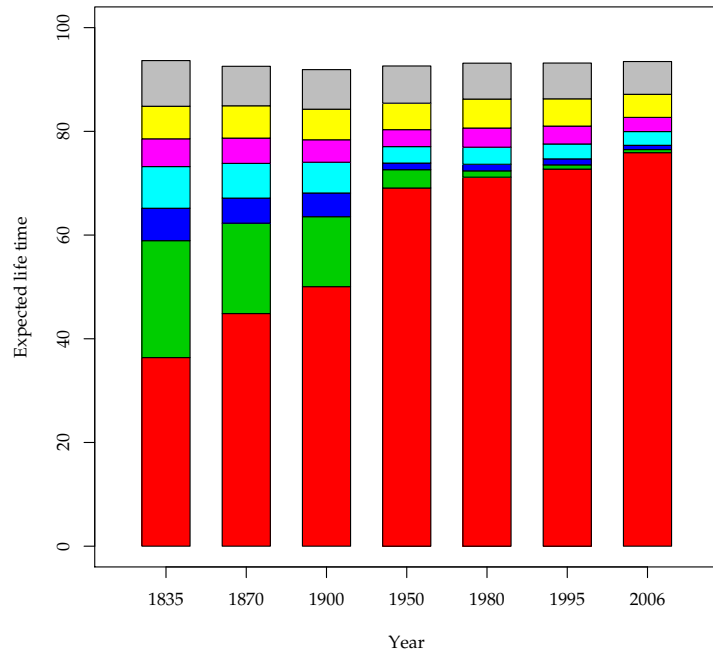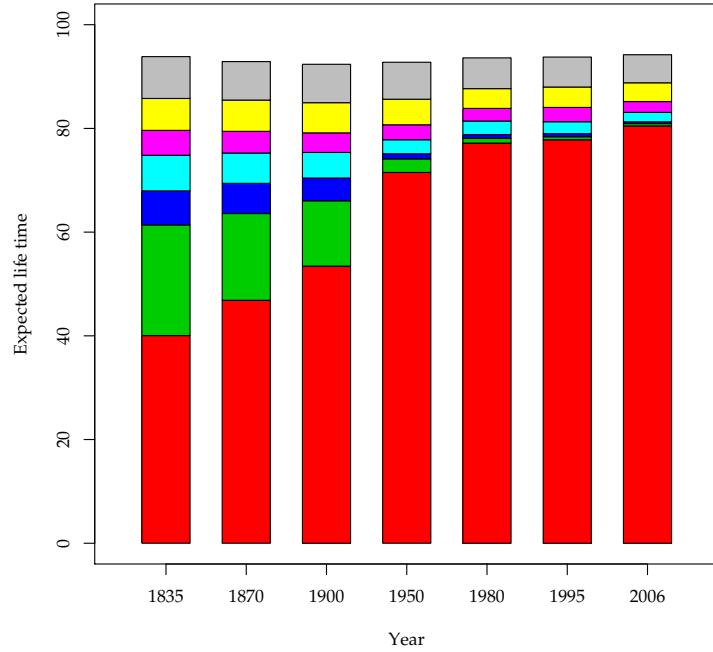
FIGURE 3. Expected total life times for Danish females (top) and males (bottom) at selected years. Each column shows the expected total life time at birth (first box) and given survival to age 20, 40, 60, 70, 80 and 90 based on the period life table for that year.
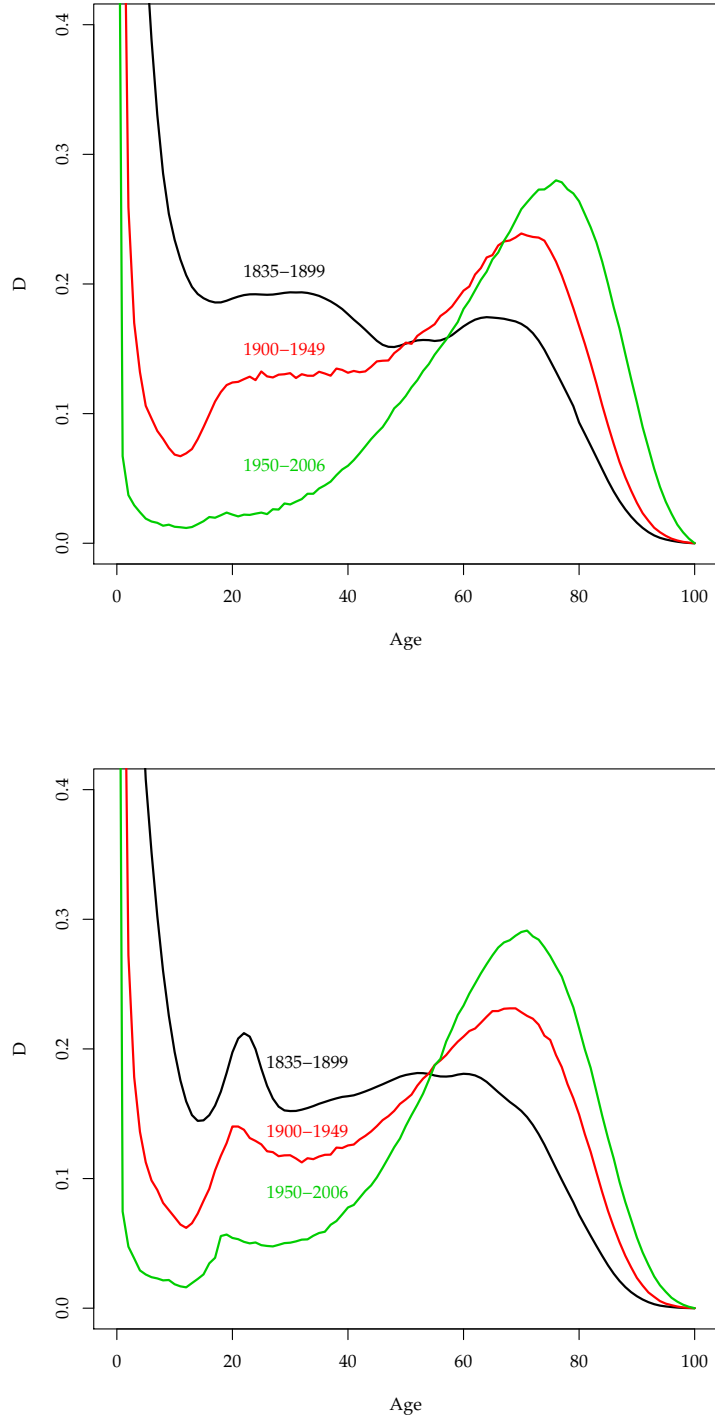
FIGURE 4. Sensitivity of expected life time at birth to improvements in age-specific death rates for Danish females (top) and males (bottom). The plot shows the average kernel, D, over the periods 1835–1899, 1900–1949 and 1950–2006; see Section 4.1 for the definition and interpretation of D. The value at age 0 for the three periods is 6.9, 4.6 and 0.8 for females and 8.0, 5.7 and 1.1 for males.
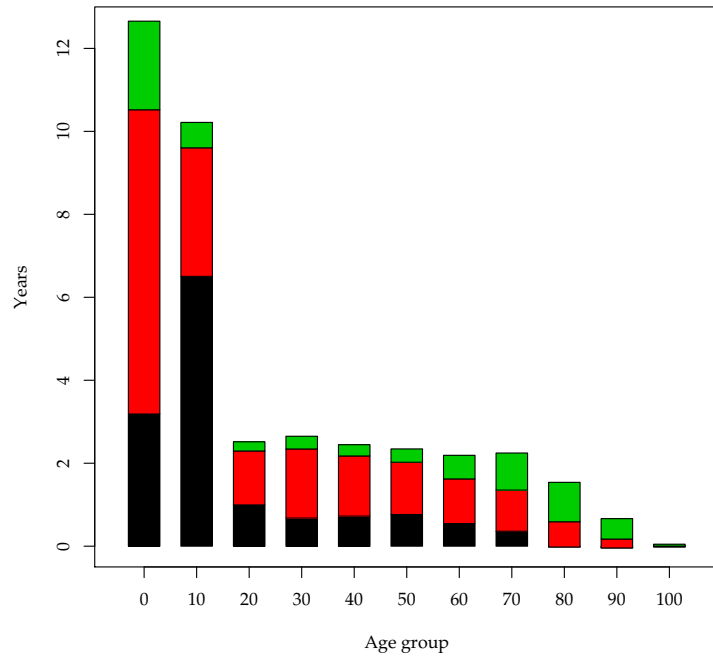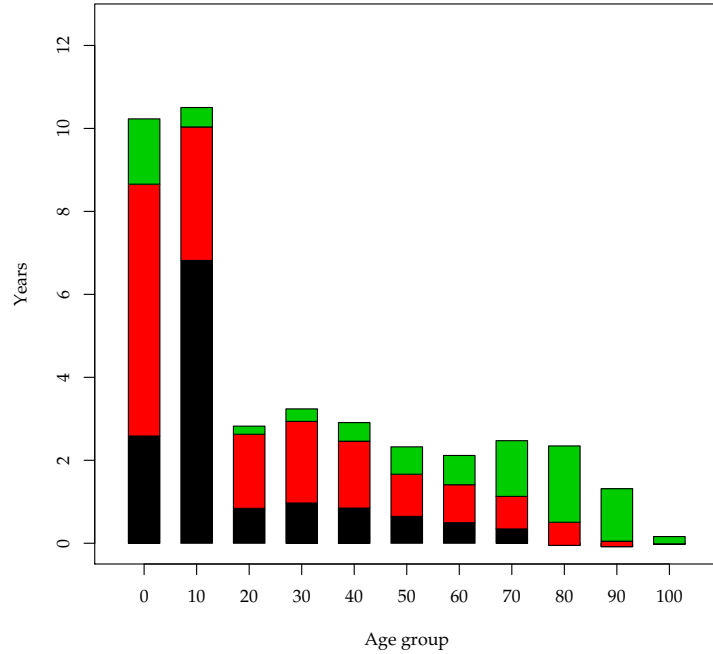
FIGURE 5. Decomposition of the life expectancy gain from 1836 to 2006 for Danish females (top) and males (bottom). The plot shows the life expectancy gain attributable to age groups 0, 1–10, 11–20, ..., 91–100 over the three periods 1835–1900 (bottom boxes), 1900–1950 (middle boxes) and 1950–2006 (top boxes); see Section 4.2 for details.
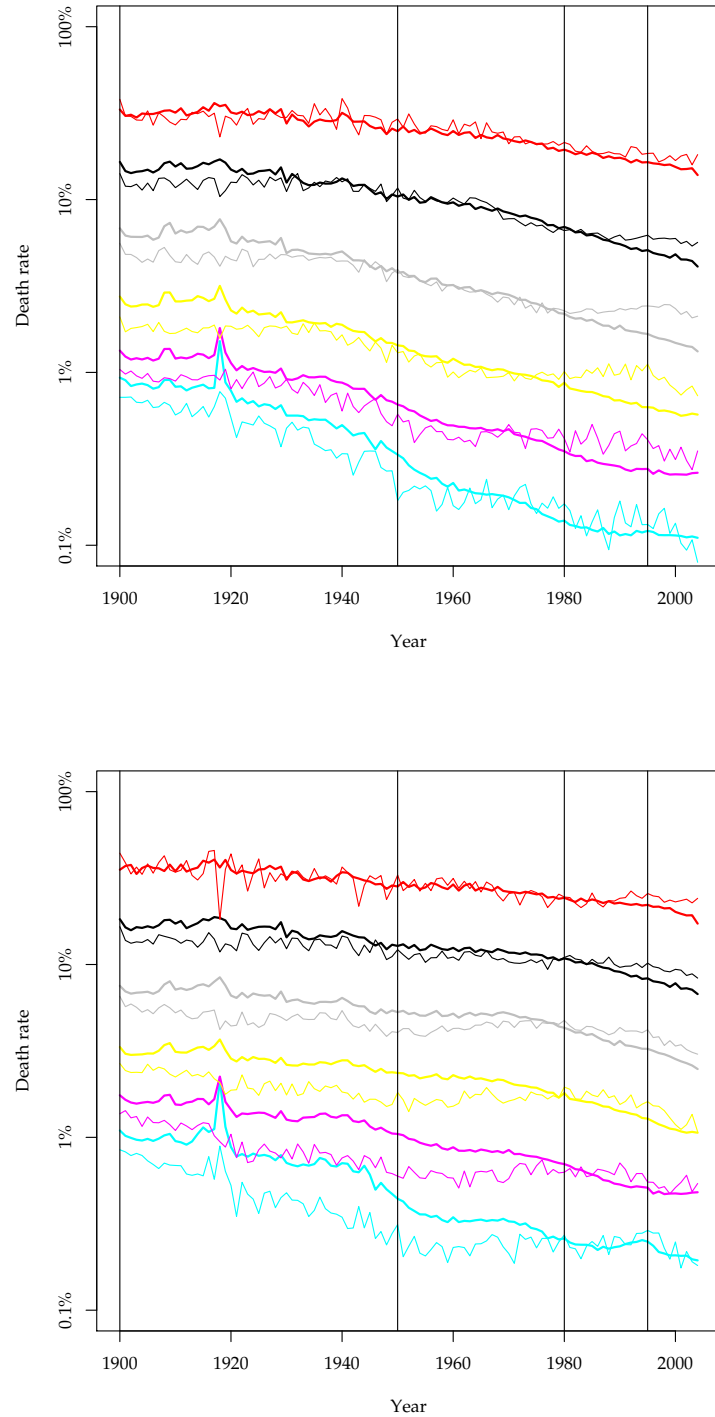
FIGURE 6. Development in Danish and international female (top) and male (bottom) age-specific death rates from 1900 to 2004 for ages 40, 50, ..., 90. Danish rates are shown with thin lines and international rates with thick lines. Top lines represent 90-year-olds and the other ages follow below in decreasing order.

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The Danish Labour Market Supplementary Pension Fund, Kongens Vænge 8, DK-3400 Hillerød, Denmark