A Balanced Factor Approach to Investing

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Abstract

Portfolio construction based on risk allocation principles and diversification forms the core of ATP's investment management. The fundamental belief is that a properly diversified portfolio levered to an acceptable level of risk is the best path to deliver the required expected return over time. This paper outlines our factor investment framework: the top-down spanning of the investment universe on a set of investable risk factors leading to the formation of a balanced factor reference portfolio based on quantitative analyses and qualitative judgement. This framework helps us to obtain clarity, simplification and measurability of the investment process.

This paper is number one in a series of four papers on ATP's investment management. The other three papers are: Dynamic Management of Portfolio Risk in a Separated Portfolio with Liability Hedging (Mads Gosvig and Morten T. Kronborg (2), 2019), Local Factor Dynamics and the Production of Positive Portfolio Convexity (Mads Godvig and Morten T. Kronborg (3), 2019), and Factor Investment Implementation and the Rationale for Alpha and Beta Separation (Mads Gosvig and Morten T. Kronborg (4), 2019).

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1. Introduction

For more than a decade ATP's investment portfolio has (1) been based on risk and not capital, (2) been constructed in a way where no specific asset class dominates the overall risk, (3) included insurance via options to protect against tail-events and (4) been sized dynamically to ensure an appropriate level of risk given current reserves. In this paper we unfold our current understanding of the diversified risk-engine, called *Balanced Beta+*, laying the ground for our investment portfolio.

For an asset owner the strategic asset allocation decision is a major return driver (see Brinson et. al. (1991)). The beliefs leading to the decision, the conceptual framework under which the portfolio is viewed, and the practical ability to perform the allocation decisions are crucial to the resulting asset allocation. As said by Markowitz (1952):

"*The process of selecting a portfolio may be divided into two stages. The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio."*

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Our fundamental investment beliefs are:

1) A positive relationship between investment risk and investment returns. An investor requires more return from an investment the more the investor perceives the investment risky. However, not all investors perceive investments risk the same way.
2) The rewarded risk can be attributed to a small set of risk factors. These risk factors span the risk across the investment universe and over time. Managing these risk factors is the most important task to obtain diversification and stable returns.
3) Asset returns are stationary and we can infer information from the history using statistical tools. However, we include a sceptical view on investment determinism and the ability to calculate ‘‘optimal” allocations based on return-statistics. Investment indeterminacy and structural uncertainty make the strategic focus more about getting the risk picture ‘‘roughly right” than being ‘‘precisely wrong” on the return expectations.

We represent risk by return variation. After all it is difficult not to make the connection between loss of capital and lower utility of the pensions. However other metrics may also be relevant (e.g. environmental impacts). Use of return variation, and co-variation, allows us to apply our beliefs on a broad and reasonable well-known stock of data in a structured fashion.

In relation to the practical ability to execute the asset allocation decision the main prerequisite is the use of leverage on different levels of the portfolio. The ability to decouple the cash allocation from the risk allocation expands the set of relevant allocations and makes the risk allocation the operative centre of the allocation decision.

In this paper we present our approach to constructing a portfolio in an agnostic risk factor approach with the factor loadings aiming for a balanced risk profile over time. We restrict ourselves to a “static” portfolio approach since we see it as a sufficiently simple starting point to form a portfolio construction view. Static in this sense is not “buy and hold” but rebalancing to a static risk profile which may imply substantial trading activities. More specifically: In Section 2 and 3 we discuss how to decide on a set of risk factors. In the main body of the paper, Section 4, we present a multi-dimensional balanced factor reference portfolio called *Balanced Beta+* and discuss the considerations leading us to the portfolio mix. In Section 5 we illustrate the production of factor loadings through actual investments. Section 6 concludes.

Our Balanced Beta is the central part of our investment process which is a dynamic process based on our beliefs, quantitative methods and industrial strength in operating an institutional portfolio.

2. A selection of factor asset allocation schools

We are aiming for a balanced risk profile in the cross-section of factors. Static loadings to risk factors are chosen targeting a balanced risk profile and avoiding concentration risks to single factors. This is in line with the ‘‘risk-parity” approach (see e.g. Asness et al. (2011), Qian (2011) and Chaves et al. (2015)) and differs substantially from mean-variance optimization and similar approaches within modern portfolio management (see Blyth et al. (2016) and the FIFAA investment approach suggested by Harvard Management Corporation for an asset allocation literature review).

The starting point for the Balanced Beta framework is to decide on the factors spanning our investment universe. Whereas the definition of asset classes is usually straight forward, factors can be based on for example; (a) investable factors (e.g. assets, indices, financial derivatives etc.); (b) macro factors (e.g. growth, inflation, macro uncertainty, central bank liquidity); (c) statistical factors (principal-component-analysis, cluster analysis or other statistical methods). All three factor-approaches share the same methodology, namely that

investments are decomposed into loadings to the factors and the aggregated factor loadings, the ‘‘ingredients” of the portfolio, are the resulting factor risk allocation. On top of that one may get ‘‘unexplained’’ risk (risk not explained by the factors) that one can worry about and/or try to limit.

We use investable risk factors as discussed in Section 3. This puts us in line with the approach of BlackRock (see Bass et al. (2017)), where portfolios of selected assets represent factors. The BlackRock factors are listed in Table 1.

Name	Description	Representation
Economic Growth	Risk associated with global economic growth	Broad-market equity index returns over cash
Real Rates	Risk of bearing exposure to real interest rate changes	Inflation-linked bond returns over cash
Inflation	Risk of bearing exposure to changes in nominal prices	Return of ‘long nominal bonds, short inflation linked bonds’-portfolio
Credit	Risk of default or yield spread widening	Return of ‘long corporate bonds, short nominal bonds’-portfolio
Emerging Markets	Risk that emerging sovereign governments will change capital market rules	Basket of EM equity, EM CDX and EM FX
Commodity	Risk associated with commodity markets	Weighted GSCI Commodity index returns

Table 1: BlackRock factors as in Bass et al. (2017).

In this framework currency risk is also stated as a risk factor in the sense that currency risk is an important driver of portfolio volatility but not a rewarded factor.

The application of the factors on asset allocation decisions is left to the investor in the context of her specific investment objective, liabilities and constraints. Each factor defines a unique set of “bad times” or periods of underperformance. Investors must select factor exposures that cater to their comparative advantages and compensate them adequately for the accepted factor risk. From our understating this is the main key point outlined in the book by (Ang, A. (2014)).

The ‘‘All-Weather” portfolio approach of Bridgewater Associates applies investment returns on selected assets conditional on a two-state macro model. They consider time periods in which growth or inflation expectations turns out to be higher or lower than realized to form a two-factor approach. The balance of returns between macro states is at the centre of the asset allocation process aiming at minimizing the impact of macro shocks on the portfolio. For more on the ‘‘All-Weather”-approach of Bridgewater Associates see Dalio et al. (2015).

Both BlackRock’s and Bridgewater Associates’ approach to portfolio construction have been sources of great inspiration to us.

3. ATP’s investment factors

The starting point for our factor approach to investing is to decide upon the factors spanning our investment universe. Our goal is to span our investment universe, with just a few factors to obtain simplicity, in such a way that a significant level of diversification is achievable. We use investable factors. We recognize that investable factors (i.e. return streams on assets or strategies) are not the root causes of portfolio return variation. However, the advantage of this approach is the more straightforward link between factors and investments thus reducing the reliance on models linking factors and investments. Investable factors also

make us able to connect to the asset pricing literature, whereas both macro factors and statistical factors are applied less in relation to asset pricing. Furthermore, investable risk factors are representable by investment return streams with reasonably reliable data spanning +40 years paving the way for statistical analysis on a reasonably long set of data.

Factors based directly on macroeconomic data introduce a simultaneity problem as financial markets may respond based on expectations and not on realised information. In addition, macro factors are less robust over implementations due to technicalities regarding how to represent macro factors (computational details and data choices). Statistical factors have limitations in regard to stability and translation to observable events.

Returns on indices, financial derivatives and formalized long-short security strategies are natural components of investable factors. Table 2 details our top-level factors. We have deliberately kept the number of factors low. However, as explained later, each factor consists of subfactors (regional splits, different investment styles and subsets of commodities) which we use to manage the actual implementation of the factor risk.

Name	Description	Representation
Equities	Risk related to equity price variation	Broad-market equity index returns over cash
Nominal Rates	Risk related to changes in nominal yields	Major 'non-defaultable' nominal bond returns over cash
Inflation	Risk related to changes in breakeven inflation and risk related to changes in commodity prices	Return of 'long inflation linked bonds, short nominal bonds'-portfolio and long commodity futures portfolio
Other	Risk related to holding illiquid assets and liquid alternative investment strategies	Return of 'alternative risk premia'-portfolio (over cash)

Table 2: ATP's factors.

The Equity and Nominal Rates Factor span the traditional liquid long only asset class universe. However, spanning our investment universe requires more risk factors. We have included break even inflation (BEI) risk in the Inflation Factor which allows us to separate real rate risk from nominal rate risk. In other words, loadings to the Nominal Rates Factor result in a loading to real rates and (short) BEI. Subsequently, the amount of BEI risk can be controlled for in the Inflation Factor (typical by a long position in BEI). Commodities are included in the Inflation Factor as well. BEI and commodities are fairly distinct in statistical terms, but the commonality lies within the representation of inflation as 'output inflation' (breakeven inflation) and 'input inflation' (commodity prices). Never the less, over a sustained period with unexpected high inflation levels we believe both long BEI and a basket of commodity futures will perform well.

The Other Factor is represented by alternative risk premia returns. We can attribute some of the risk induced by active management of liquid assets to alternative risk factors (in the sense of Fama-French (1993)). Among factors we control for are Value and Momentum (e.g. see Asness et. al (2013)), Low Risk / Low Beta (e.g. see Frazzini and Pedersen (2014)), Carry (e.g. see Koijen (2018)) and Trend (e.g. see Moskowitz et. al. 2011).

Our factors are all based on liquid investments. We believe that illiquid investments (private real estate, private equity and private credit among others) are in general priced relative to (collections of) liquid investments with similar risk properties including a specific risk related to illiquidity. Specifically, we attribute some of the risk related to investments in illiquid assets to somewhat complex options-based strategies in liquid assets and hence achieve a liquid representation of a significant risk element in our portfolio. The representations of the risk related to illiquid investments will be discussed in Section 5.

Our choice of risk factors is not scientific in the sense that there is an element of hindsight information as we fit the risk factors to our investment universe. The risk factor choice seeks to bridge the academic ambition to be able to exhaust the universe without undue datamining with the practicalities of the investment world where simplification, robustness and operational limitations are necessary ingredients.

Our risk factors are linked to rewarded risk premia. The Equities Factor is related to the equity premium (expected reward for bearing earnings uncertainty). The Nominal Rates Factor is related to the real rate premium (expected reward for bearing real purchasing power uncertainty) and the inflation premium (expected reward for bearing inflation uncertainty). The Inflation Factor consist of a short position in relation to the inflation premium (long BEI) and long position in relation to the commodity future premium (expected reward for bearing commodity price uncertainty due to a hedge demand by commodity suppliers). The Inflation Factor itself may not carry a significant, if any, positive premium consistent with the historical returns, but we strongly believe it to have great diversification properties in relation to the Equities Factor and the Nominal Rates Factor.

Currency risk is not a risk factor in our factor framework. This is due to a long-time investment policy to hedge currency risk out of all investments and thus separating the currency position choice from the actual investment. In general, static currency positions have not consistently added value neither in terms of return nor diversification. However, loadings to active trading strategies in currencies (e.g. trend and carry) are applied in the Other Factor.

It is important to emphasize that our aim is to span our investment universe with factors in such a way that a significant level of diversification is achievable from the factor allocation decision. Residual risk not spanned by our factor approach also exists and should be managed as well. We discuss this in detail in (Mads Gosvig and Morten T. Kronborg (4), 2019). Our aim is to map actual investments to the risk factor representation and thus enable management of the aggregated loadings to the risk factors. But first we present and discuss the properties of a reference portfolio with appropriate loadings to the risk factors called *Balanced Beta*.

4. A balanced factor reference portfolio

A first application of the factor framework is to establish a factor reference portfolio functioning as an investment anchor. This is a crucial strategic element for a total return fund. It is a focal point for the set of factor loadings, and the ‘‘risk free” exposure around which the organization can take initiative. Long-term investors and other total return funds should agree with their sponsors on this factor reference. In that respect expectation management becomes the starting point for fund management. We refer to Ang (2014) chapter 14 for further discussion on the importance of this aspect. In this section we define a balanced factor reference portfolio, *Balanced Beta+*, via a multidimensional vector of factor loadings.

- **Defining the factors**

We select the constituents of the balanced factor reference portfolio in a 2-level process combining the main risk factor representation with diverse regional representation for equities, high yield spreads, nominal rates and breakeven inflation and across types of commodities and investment styles. The building blocks of our balanced factor reference portfolio are (for reference see Table 2):

- The Equities Factor is comprised of five sub-factors representing broad equity indices across major regions (US, Europe, Asian) as well as liquid credit spread indices (US and Europe). The latter represent equity-related risk in a slightly different form, but we believe that the main driver for equity and credit spread returns is the same.
- The Nominal Rates Factor consist of two sub-factors which represent government bonds in medium term (7-10 year) maturity in US and Europe.
- The Inflation Factor consist of four sub-factors representing input inflation (gold and industrial commodities) and output inflation (US and Europe breakeven inflation[4]).
- The Other Factor is grouped into three sub-factors of alternative risk premiums (ARP) / trading strategies, which are all 100 percent quantitative based:
  a) *Directional*: Long or short positions in stocks, bonds, FX forwards (G10 and EM) and commodities (oil and a selection of hard commodities) based on Time Series Momentum signals, and long or short positions in FX forwards and commodities based on carry signals[5].
  b) *Macro Factor Rotation*: Long or short positions in stocks and bonds based on whenever the economy is believed to be in expansion, late cycle, recession or recovery.
  c) *Relative*: Beta neutral stock portfolios based on momentum and value signals, and beta neutral bond portfolios based on momentum and carry signals[6].

For stocks we trade in US, Europe and Asia and for bonds we trade in US and Europe. All trading strategies are hypothetical returns gross of any trading costs.

Below we define two factor reference portfolios; *Balanced Beta* and *Balanced Beta+*. The former is a three-factor portfolio with loading to the Equity, Rates and Inflation Factor, and the latter adds a loading to the Other Factor. Balanced Beta is a reference factor portfolio loading to the more traditional long-only premia, whereas Balanced Beta+ is our best total return factor portfolio applying alternative long-short and timing strategies as well.

- **Allocating risk not notionals**

Before defining and illustrating the two factor reference portfolios we repeat that allocating risk and not notionals is the natural starting point for ATP. For ease of reference we introduce the industry standard reference portfolio given by a 60/40 percent *notional* split between equities and bonds. More specifically, in Figure 1 the portfolio *Equity-Rates 60/40 Notionals* is a 60/40 percent split between the Equities Factor and the Nominal Rates Factor. This portfolio, with monthly rebalancing, has realized 8.76% annualized volatility over the total period 1973-Q2 – 2018-Q4.

We allow for leverage and, consequently, comparisons between portfolios need to be done at a *per risk basis.* If for example (for some reason) the desired target risk for the total portfolio is equal to the risk achieved by a 60/40 equity-rates notional split, we can meaningfully compare the *Equity-Rates 60/40 Notionals* to other portfolios with the same target risk for the total portfolio.

[4] Break-even-inflation returns are produced by long positions in inflation linked bonds and short duration matched positions in nominal bonds. Before the existence of the linker market (1997 for US and 2009 for Europe) results are based on hypothetical inflation linked bond returns provided by Bridgewater Associates LP.
[5] From March 1973 to 1998 for commodities TSMOM and carry, and from March 1973 to 2002 for FX TSMOM and carry, we use returns provided by AQR Capital Management.
[6] From March 1973 to 2005 we use returns provided by AQR Capital Management.

In Figure 1 the portfolio *Equity-Rates 60/40 Notionals Targeting Risk* targets at any point in time an ex-ante annualized volatility of 8.76%, i.e. equal to the average risk in *Equity-Rates 60/40 Notionals* over the full period, using a 10-year trailing data window for risk estimation. We observe that to keep the ex-ante risk at 8.76% volatility the notionals must be adjusted to levels between roughly 80%-115% depending on the point in time. Looking at performance the rather slow (10-year rolling data window used to estimate risk) adjustment of expected risk seems to have little influence on total returns. However, we see rather bad timing related to reducing notionals in the aftermath of the tech-bobble-bust in 2001 and the Global Financial Crisis in 2008-2009 thereby missing out on the equity rebounds relative to the *Equity-Rates 60/40 Notionals*.

In the following we use the *Equity-Rates 60/40 Notionals Targeting Risk* portfolio as reference since all other portfolios will be risk controlled in the same manner for fair comparisons.

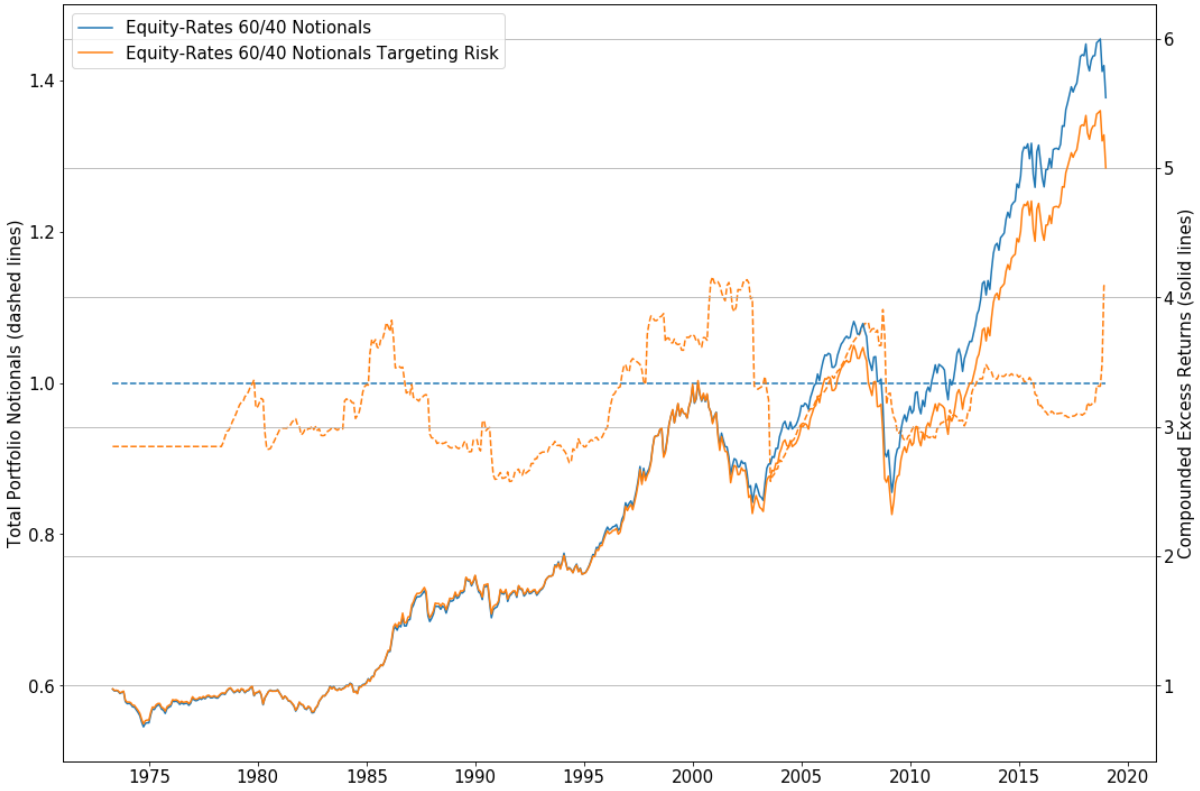


Figure 1: Right y-axis: historical monthly compounded DKK (fully hedged) returns excess of funding. Left y-axis: total portfolio notionals. Blue lines: portfolio where 60 and 40 percent of the notionals are allocated to the Equities Factor and Nominal Rates Factor, respectively, and rebalanced monthly. This results in an annualized volatility of 8.76% over the period. Yellow lines: the same portfolio except that the ex-ante risk is controlled to be 8.76% at each monthly rebalancing point using a 10-year trailing window for risk estimation.

- **Balanced Beta**

In Figure 2 we illustrate historical monthly compounded excess returns (fully currency hedged and converted into Danish Kroner). The portfolios are rebalanced monthly to the relative risk weighting scheme and to an overall ex-ante risk of 8.76% annualized volatility using only past data (the trailing 10 years), thereby trying to replicate the total risk of the *Equity-Rates 60/40 Notionals* portfolio in real time.

First, observe from Figure 2 that the *Equity-Rates 60/40 Notionals Targeting Risk* and the Equities Factor has very similar patterns (full sample correlation is 94% and never below

86% in any 5-year subperiod). Although not recognized by many investors this is simply due to equities being around three times as risky as medium-term bonds.

This brings us to the factor reference portfolio, *Balanced Beta*. The main aim of the portfolio is to achieve a significant level of diversification based on the historical returns and to some extent beliefs about the future. Our starting point is an *Equity-Rates Risk Parity* portfolio, i.e. an equal risk split between the two major and distinct return drivers; the Nominal Rates Factor and the Equities Factor. This portfolio is in line with the modern literature on risk-based asset allocation. As illustrated by Figure 2 this portfolio delivers much more return than the *Equity-Rates 60/40 Notionals Targeting Risk* portfolio, at the same (ex-ante) risk. This result is primarily driven by the diversification property between the Equities Factor and the Nominal Rates Factor. All though varying over time the correlation of the two factors is approximately zero over the total period. To obtain the same amount of risk (ex-ante) we lever up the *Equity-Rates Risk Parity* portfolio. Alternatively, one could have obtained the same return with less risk.

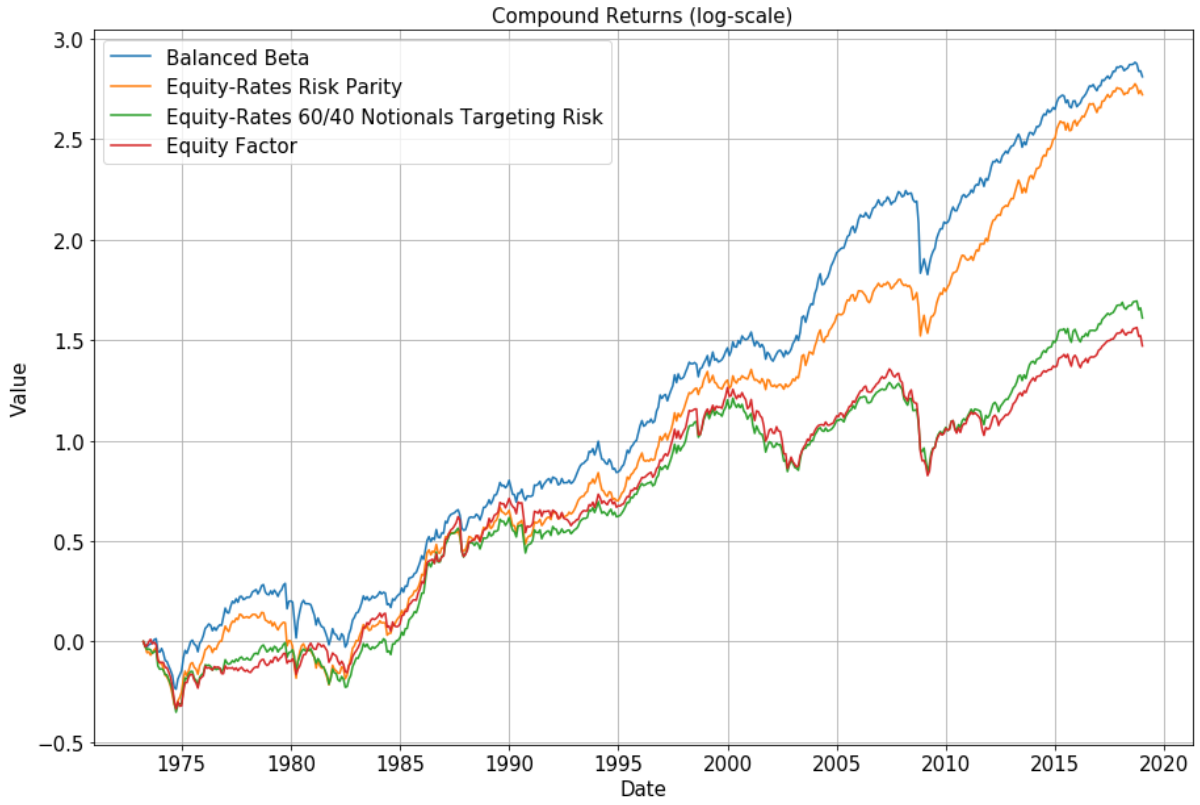


Figure 2: Historical monthly compounded DKK (fully hedged) returns excess of funding from ex-ante 8.76% volatility portfolios, using a 10-year window for monthly risk sizing and monthly rebalancing to absolute and relative risk targets.

Relocating risk to the Inflation Factor adds significantly to the portfolio properties. The Balanced Beta portfolio is also illustrated in Figure 2. It is based on a relative risk weighting scheme which sums to 40% to the Nominal Rates Factor, 40% to the Equities Factor and 20% to the Inflation Factor (choice of relative risk weighting to be discussed below). As seen in Figure 2 *Balanced Beta* delivers even more return per risk than the *Equity-Rates Risk Parity*. Again, this is primarily due to diversification properties of the Inflation Factor in relation to the Equities Factor and the Nominal Rates Factor. To obtain the same amount of ex-ante risk we lever up the portfolio once more. We recognize that the portfolio, compared to the *Equity-Rates 60/40 Notionals Targeting Risk* portfolio, has benefitted from falling yields over the period (wind fall gains) but this is a minor effect compared to the diversification benefit (our calculations shows that only around 1/3 of the returns from the Nominal Rates Factor is due to falling yields, the rest is due to ‘carry’ and ‘rolling’ down the yield curve).

- **Leverage**

The primary reason that *Balanced Beta* achieves more return per risk is due to diversification. However, at the portfolio level total risk per notional decreases dramatically. For the portfolio to achieve the same ex-ante risk, 8.76% annualized volatility, the portfolio needs to be levered. In capital terms, the diversification benefit from the historical factor-correlation structure makes the *Balanced Beta* implementation a low-volatile portfolio with approximately 3.5% annualized volatility. The three underlying fundamental factors are riskier; the Nominal Rates Factor has around 4-6% volatility, the Equities Factor around 10-15% volatility (a relatively high implicit capital allocation to credit spreads reduces the volatility) and the Inflation Factor around 7-9% volatility (commodities much higher, BEI much lower). In capital terms DKK 60 of the Nominal Rates Factor, DKK 25 of the Equities Factor and DKK 15 of the Inflation Factor makes up the *Balanced Beta* portfolio with a volatility around DKK 3.5.

We recognize that not all investors can apply leverage and that the excess return from the portfolio is partly due to leverage constraints. Since *Balanced Beta* substantially overweights low risk factors in notional terms, compared to the market weighted portfolio, it potentially benefits from the "low risk / low beta - risk anomaly", which refers to the fact that low risk / low beta assets historically have outperformed high risk / high beta assets on a risk adjusted basis. For references see Frazzini-Pedersen (2014, 2012) and Asness-Frazzini-Pedersen (2012) who provide empirical evidence across and within countries and markets. The rather superior returns of *Balanced Beta* are most likely due to the low risk anomaly. To some extend we reckon that this is an example of an institutional advantages of ours since not all investors are capable of (or comfortable with) the amount of leverage needed to obtain adequate (high enough) expected returns. For a discussion on leverage restrictions, aversion and asset pricing implications we again refer to Frazzini-Pedersen (2014, 2012) and Asness-Frazzini-Pedersen (2012).

- **How to decide upon the factor risk mix**

Other permutations of the factors than the 40-40-20 percent risk loadings have been analysed. They obviously result in different outcomes and may prove more attractive over sub-periods and/or the total period. However, the simplicity of the combination appeals to our sceptical view on precision inferred from historical data. The strategic Balanced Beta portfolio turns out to be close to the optimal mean-variance portfolio when evaluated over the total period. This has, however, never been the argument for setting the factor loadings in the first place. Rather the starting point has been the *Equity-Rates Risk Parity* portfolio in line with the modern literature on risk-based asset allocation. Admittedly, considering only two factors for a start eases the allocation decision. For example, in the presence of only two factors we note that even marginal risk contribution to the factors is in line with the Equal Risk Contribution portfolio (see e.g. Maillard et al. (2010)) and the Most Diversified portfolio (see Choueifaty and Coignard (2008)). The allocation weight to the Inflation Factor is partly based on macro considerations and beliefs about the future and is needed exactly when both the Equities Factor and the Nominal Rates Factor are producing bad returns, as discussed below.

- **Diversification and macro states**

The Inflation Factor adds significantly to portfolio properties. It contributes positively to returns in periods where the Equity-Rates Risk Parity portfolio performance badly. This is most pronounced in the high-inflation period of the '70s. This is evident from Figure 3 where we

observe that the *Balanced Beta* portfolio builds up a lead over the first 10 years compared to the *Equity-Rates Risk Parity* portfolio (the 1983 trailing 10-year geometric return is 2% versus 0.5%). The lead grows substantially again between the 2001 dot-com-bubble burst and the 2008 Global Financial Crisis (although a great period for the *Equity-Rates Risk Parity* portfolio) and is mainly due to the skyrocketing commodity prices of that time (our commodity futures basket realized an ex-post Sharpe-Ratio of 1.6 over 2002-Q1 – 2008-Q2). It is furthermore remarkable that despite the prolonged negative returns in break-even inflation historically, see Figure 4, during the repricing to stable low inflation-expectations in the late 1980's and 1990's, the Inflation Factor has added to both returns and portfolio diversification over the period in consideration.

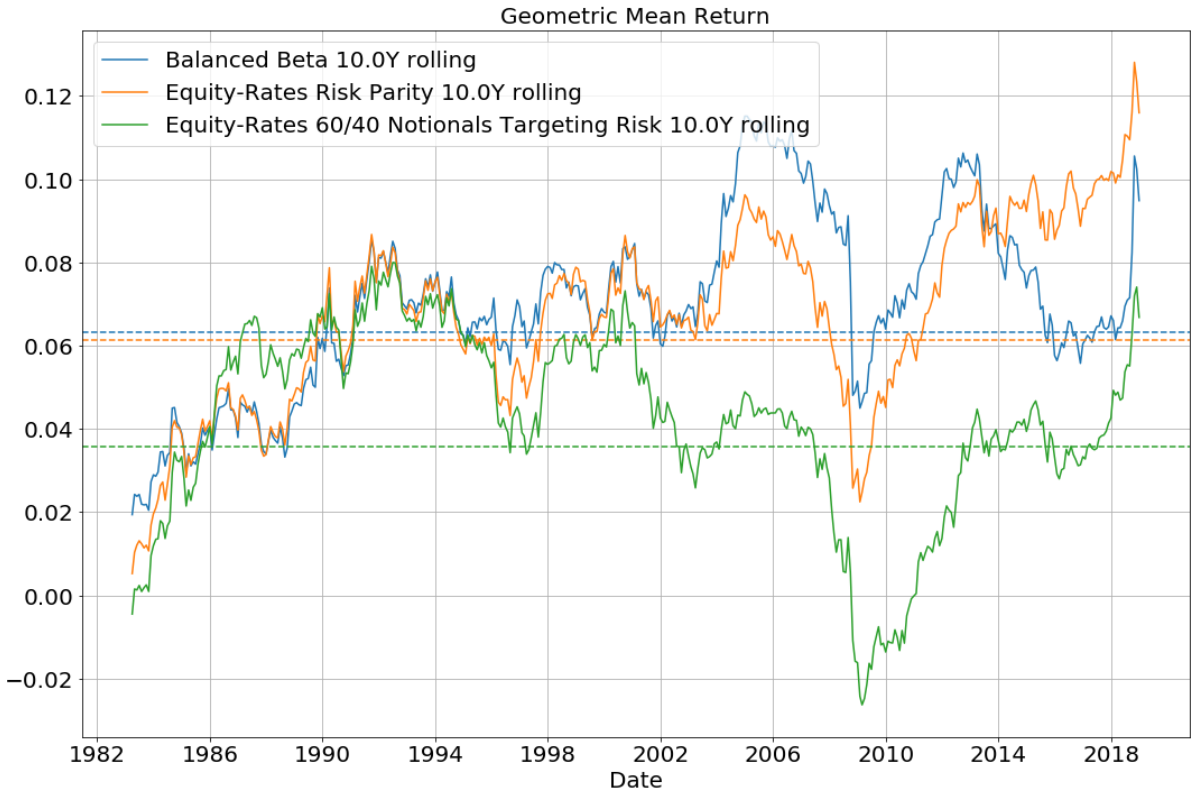


Figure 3: 10-year annualized geometric means of DKK (fully hedged) returns excess of funding for portfolios with 8.76% ex-ante volatility, rebalanced monthly.

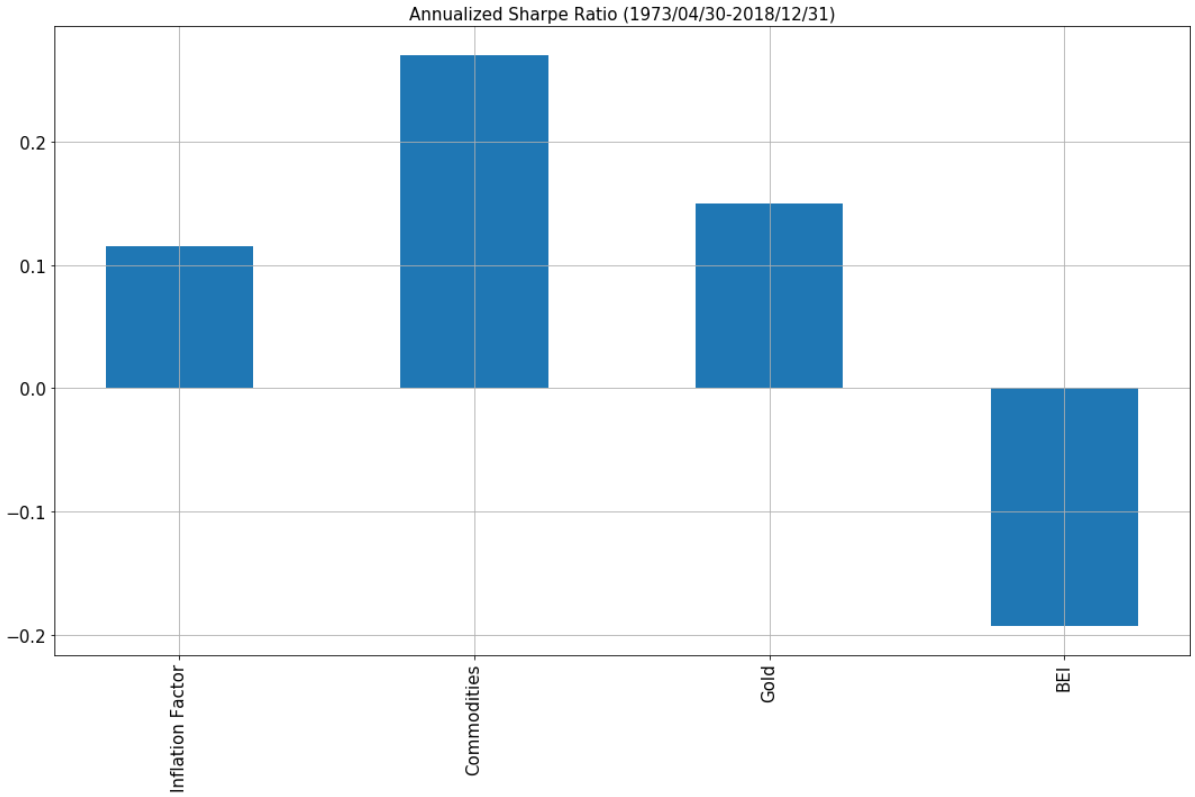


Figure 4: Realized ex-post Sharpe Ratios of the components making up the Inflation Factor of Balanced Beta.

The value added by the Inflation Factor also materializes in the ability to apply allocation decisions to improve portfolio properties based on conditional macro-states. Inspired by Bridgewater's All-Weather approach we recognize that the three factors constituting the Balanced Beta portfolio have different biases to growth and inflation shocks, the two main macro factors commonly considered to explain asset pricing variation.

More specifically we show in Figure 5 for a US version of the three factors, using US inflation (CPI) and growth (GDP) data, that the Equities Factor and the Nominal Rates Factor have opposite bias to growth shocks and the same bias (direction) to inflation shocks. If the magnitude of bias to the two environmental states are roughly equal, and growth and inflation shocks appear roughly with the same size and frequency, the two factors will (all else equal) be zero correlated, as is the case historically. The Inflation Factor is the only factor with a positive bias to inflation-up shocks. From a diversification point of view the Inflation Factor fits perfectly into a blend of the Equities Factor and the Nominal Rates Factor. Based on this simplistic approach we should allocate slightly more to the Inflation Factor than to the Equities Factor and Nominal Rates Factor to be balanced around shocks in both directions regarding the two environmental macro states. However, taking into consideration that the monetary experiments of the '70-ies may not be replicated and hence future growth and inflation rate volatility (and covariation) may be different, and given our expectations for the magnitude of return per risk for the assets contained in the Inflation Factor versus the assets contained in the Equities Factor and the Nominal Rates Factor, we decide to allocate half the risk to the Inflation Factor (hence the 40-40-20 risk split).

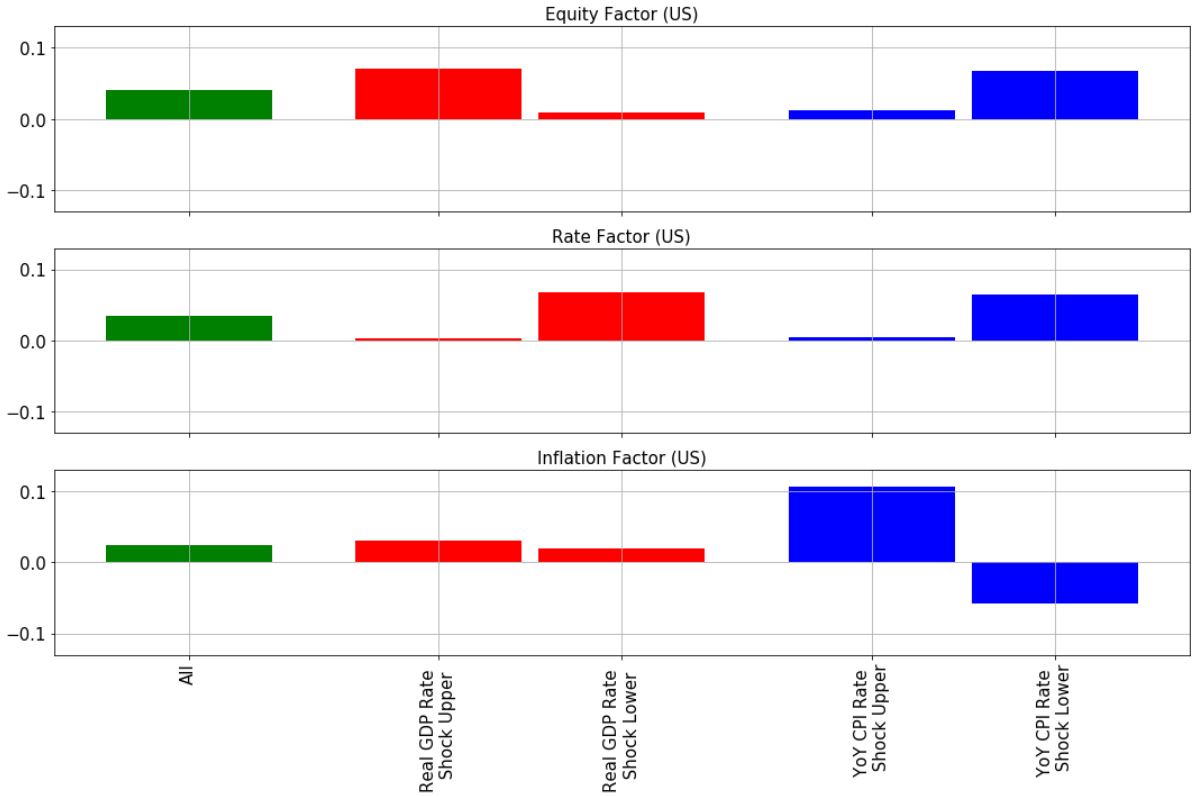


Figure 5: Factor returns conditional on macro shocks being in the 'upper' or 'lower' part (above or below median value). We show annualized mean DKK (fully hedged) returns excess of funding for factors with 8.76% ex-ante volatility, rebalanced monthly. Data period 1973-Q3 – 2018 using quarterly time periods. US data only, i.e. US macro data and US versions of the three factors. Macro shocks are simply given by current value minus average over trailing 4 quarters divided by the full period standard deviation.

- **Balanced Beta+ (adding the Other Factor)**

As described the Other Factor is represented by directional strategies, macro factor rotation strategies and relative strategies. We trade stocks, bonds, commodity futures and FX forwards contracts to achieve the desired risk exposures. The number of strategies is kept low to maintain simplicity. The marginal benefit from adding another strategy to an already diversified pool of strategies is small and decreasing.

We defined the *Balanced Beta+* portfolio by subtracting 5% (in absolute terms) from the relative risk exposures to the three factors of *Balanced Beta* (the Equities Factor, the Nominal Rates Factor and the Inflation Factor) and adding a 15% relative risk allocation to the Other Factor. The relative risk within the Other Factor are 40% to directional strategies, 10% to macro factor rotation, and 50% to relative strategies, thus obtaining roughly a 50/50 split between beta-timing and beta-neutral strategies. Within each type of strategy, we apply equal risk among types/styles and regions. This minimizes the risk of data mining / optimization during the risk allocation process.

The standalone return per ex-ante risk of the Other Factor is even higher than *Balanced Beta*. Consequently, in Figure 6 we illustrate both *Balanced Beta+* and *Balanced Beta+ Adjusted*. The latter is adjusted such that the Other Factor has the same return per risk as *Balanced Beta*. The excess returns from *Balanced Beta+ Adjusted* (over *Balanced Beta+*) is therefore by construction purely a diversification benefit from adding a 15% loading to the Other Factor.

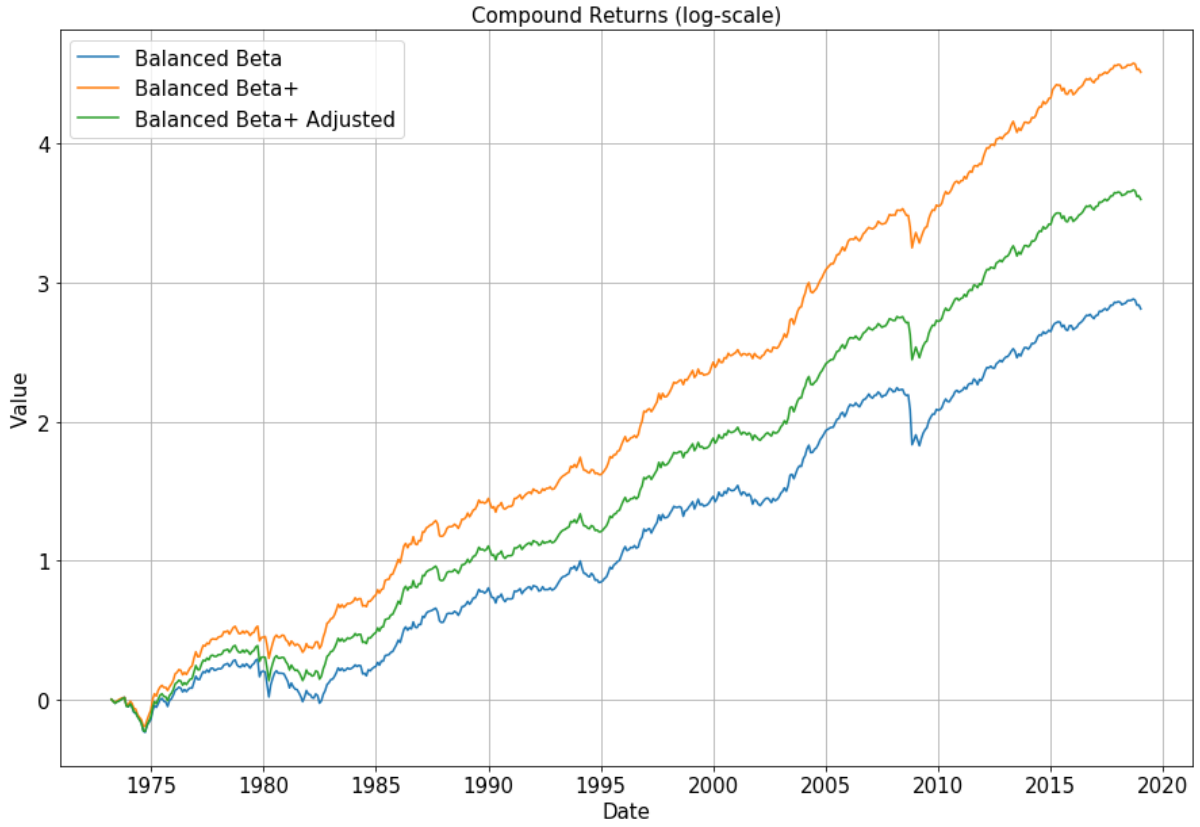


Figure 6: Historical monthly compounded DKK (fully hedged) returns excess of funding from ex-ante 8.76% volatility portfolios, using a 10-year window for monthly risk sizing and monthly rebalancing to absolute and relative risk targets.

The Other Factor, and the three trading styles (grouping of ARP-factors) defining the Other Factor, are close to uncorrelated to both the Equities Factor, the Nominal Rates Factor and the Inflation Factor. The same is the case for the trading strategies themselves. Correlations are given by Figure 7. Also, the Other Factor is close to invariant to the two macro shock states in Figure 5 (not illustrated here but see Illmanen et. al. (2014) for illustrations for value, momentum, carry, defensive and trend factors). The Other Factor contributes conveniently in scenarios where *Balanced Beta* is generating losses. The very design of the Other Factor is exactly to be market neutral and thereby, hopefully, generate positive returns in those scenarios. Sportingly we observe by Figure 6 that the Other Factor seems to add value consistently over time, without being better or worse off in any particular sub-period (the distance between the two graphs on the log-scale is almost monotonically increasing).

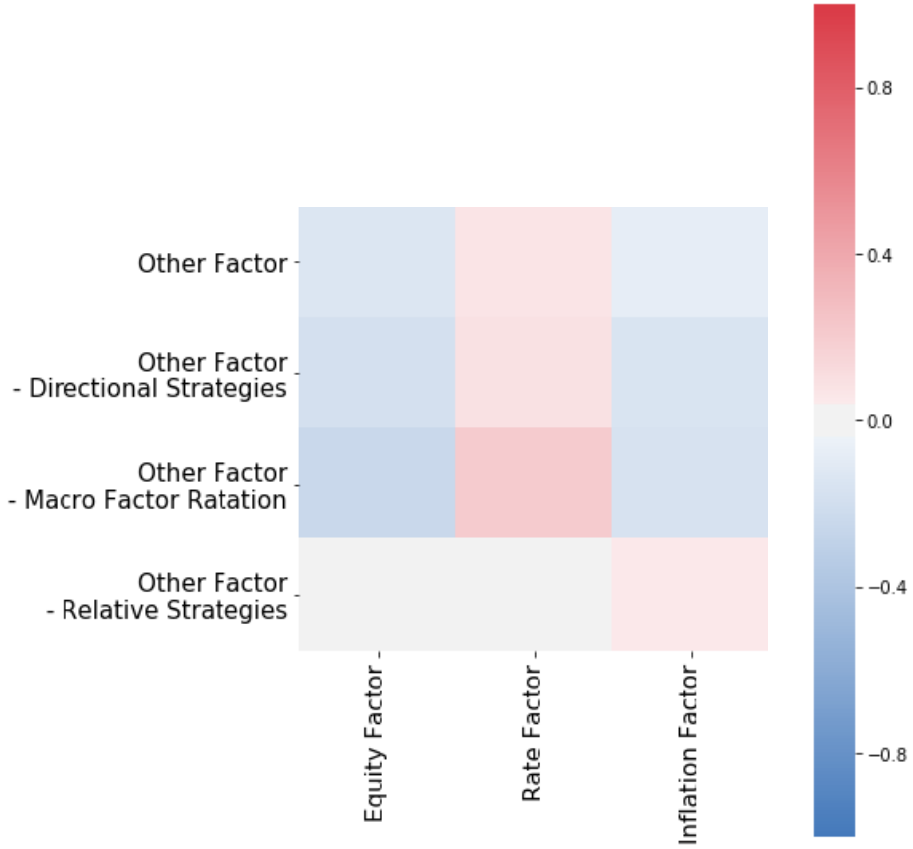
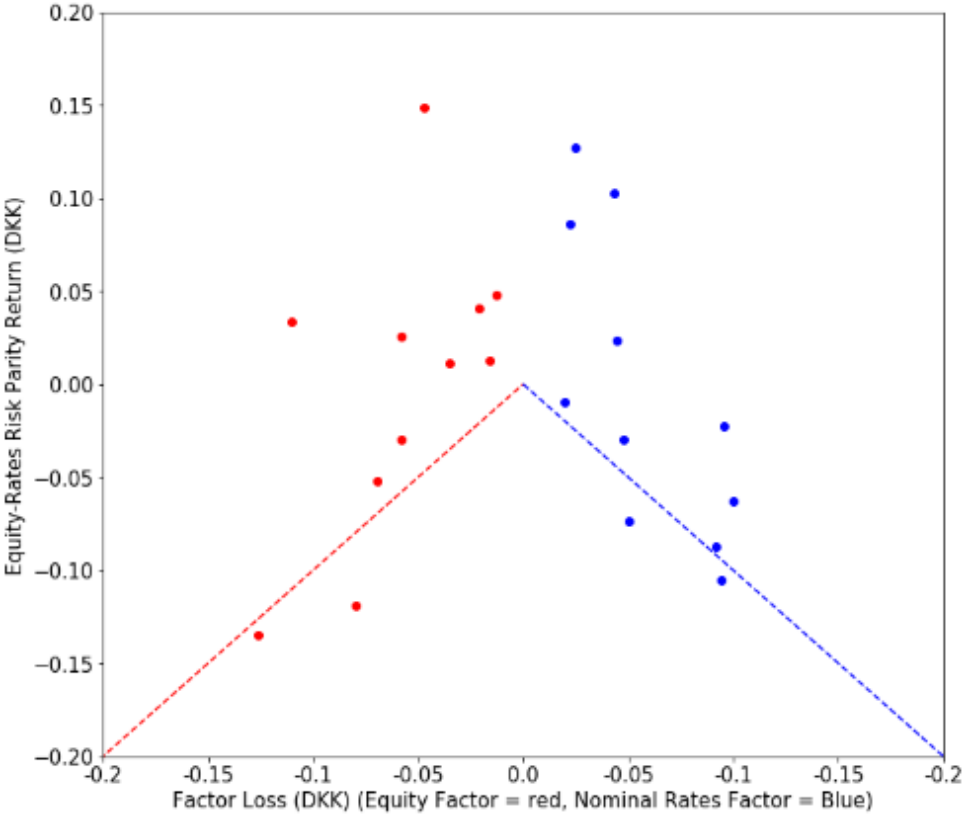


Figure 7: Empirical correlations between Factors and trading styles (grouping of ARP-factors) defining the Other Factor.

- **Diversification and risk off**

What we cannot clearly see in Figure 2 and Figure 6 is how the Inflation Factor and the Other Factor are performing when one or both of the main factors, the Equities Factor and the Nominal Rates Factor, realize large losses. The main argument for loadings to the Inflation and Other Factor has been to partly diversify away bad Equities Factor and Nominal Rates Factor states. In Figure 8 we illustrate this in what we call the *reverse v-plot*. We consider annual non-overlapping calendar year returns, but we only consider observations where the Equities Factor, the Nominal Rates Factor, or both, generates negative returns. We plot total portfolio returns against the largest Factor loss of the Equities Factor and the Nominal Rates Factor. The two dotted lines represent visual helping lines with slope 1 and -1, respectively. In the upper window of Figure 8 we consider the *Equity-Rates Risk Parity* portfolio. First observe, that the portfolio is in fact equally exposed to losses in the Equities Factor and the Nominal Rates Factor. Second, observe that no points fall greatly under the dotted lines, indicating that severe losses are not obtained simultaneously in the Equities and Nominal Rates Factor. On the other hand, when one of the two factors are suffering greatly the other is likely not to be a real ‘‘risk off’ factor. This is a consequence of the well-known fact that the co-movement of the asset-classes comprised within each of the two factors tends to co-move more in stressed scenarios, possibly due to widening risk premiums (risk aversion) and/or decreasing liquidity. In the lower window of Figure 8 we consider *Balanced Beta+*. We observe that the Inflation Factor and the Other Factor combined seem to help in some of the extreme loss scenarios for the Equities Factor and the Nominal Rates Factor.



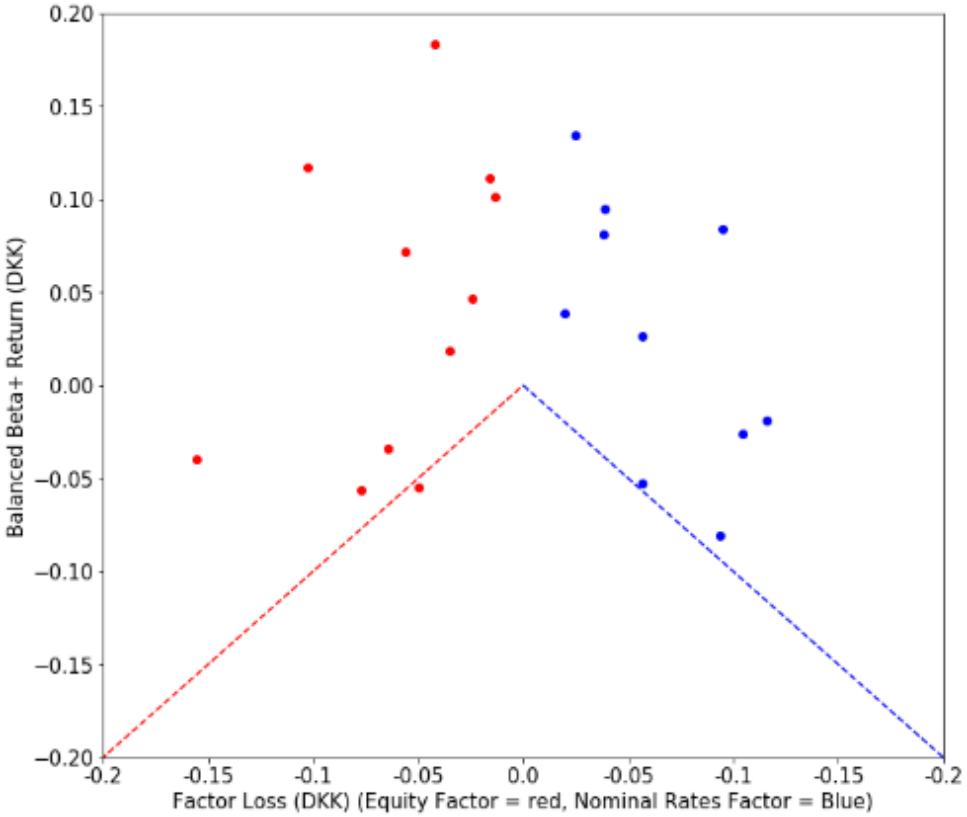


Figure 8: (The reverse v-plot): Non-overlapping calendar DKK (fully hedged) returns excess of funding from ex-ante 8.76% volatility portfolios. We only consider observations where the Equities Factor and/or the Nominal Rates Factor generates negative returns. Values on the x-axis represent the loss in the Equities (Nominal Rates) Factor given the loss is greatest in the Equities (Nominal Rates) Factor. The two dotted lines represent visual helping lines with slope 1 and -1, respectively.

Observe for instance that the biggest loss for the Equities Factor (2008) is in fact larger for *Balanced Beta+* (≈ DKK -0.16) than for the *Equity-Rates Risk Parity* portfolio (≈ DKK -0.125). This is due to *Balanced Beta+* having more Equities Factor exposure at the same portfolio level risk than the *Equity-Rates Risk Parity* portfolio, due to diversification between the four factors. However, the total portfolio loss in this scenario is significant smaller for the *Balanced Beta+* portfolio. This is driven by the Other Factor being great off in this scenario (2008). One could claim that it is pure luck. However, in general we believe the Inflation Factor and the Other Factor to be of great help in these kinds of scenarios with a significant probability, and worsening total portfolio returns with a non-significant probability.

- **The complexity of the Other Factor**

An important question is whether the diversification benefit *observed in the historical data* from the Other Factor should be monetized by increasing the sum of factor risk loadings and thereby increase the expected return for the same amount of risk (as the case for *Balanced Beta+*) or whether the Other Factor should crowd-out other factor risk whereby the same return is produced in a more factor-diversified way, i.e. with lower total risk? The answer depends on the belief in the historical partly back-tested returns from ARP (trading strategies) and to what degree they represent future returns and your (or your manager's) ability to re-produce the premia, i.e. act as assumed (in the back-tests). If you believe the family of ARP-factors are well-anchored academically, will persist in the future, and is implementable for you as an institution, the Other Factor is both an excess return-driver and a risk-diversifier, despite the complexity (see the book of Pedersen (2015) for a comprehensive survey of hedge fund strategies). If you have less trust in the robustness and longer-term existence of the ARP-factors (the factors were just ''lucky", see Harvey (2015)) and/or you cannot realistically trade the relevant markets as high-frequent, low-cost and in the size required, then the Other Factor is complexity more than anything else.

On paper the ARP-premia might seems too good to be true. However, you risk losing money in an (highly) unconventional way, sometimes over a sustained period of time. This might be the very reason that the premia exist in the first place (and have not been crowded out). To harvest the higher return in the long run we accept this. Being able to harvest the benefits of the ARP-factors is to some extend an institutional advantage. Not so much because some investors might not be operationally capable of exercising the strategies, since after all an increasing amount of external asset managers offer to do so in a much-formalized way, but probably more due to the challenging need of a very high level of organisational education and understanding of the process underlying systematic ARP-factors. This conclusion is also the primary take-away from the in-depth analysis of the active management of the Norwegian Government Pension Fund available from Ang, Goetzmann and Schaefer (2009).

We believe in a selection of ARP-premia and the 15% risk allocation to the Other Factor in *Balanced Beta+* represents our view on the observations regarding ARP-factors described above. We see them more as a return diversifier than as an excess return driver. With a careful management of the various ARP-factors we believe hidden betas to the Equities Factor, the Nominal Rates Factor and the Inflation Factor are limited and the Other Factor (represented with a balanced ARP portfolio) is rather uncorrelated to the simple three-factor model of *Balanced Beta*. As documented in Frazzini and Pedersen (2014) the leveraged ARP-factors carry volatility and illiquidity risk which should be taken into account.

5. A balanced production of factor betas

Having established our factors and a set of reference loadings to the factors (*Balanced Beta+*) we now turn to an "actual" portfolio and how to construct a portfolio based on risk factors which has a balanced risk profile over time.

To illustrate how our factor approach is applied we consider a number of investments in different *asset classes*. The key drivers of risk for each asset class are the loadings to the risk factors. We apply a simple mapping methodology based partly on statistical analysis of historical returns of representative investments and partly on objective qualitative decompositions of investments into parts that then are mapped to the factors. The latter is primarily the case for private and more illiquid market investments such as Private Equity, Infrastructure, Real Estate and Timber, for which we have limited historical data.

We emphasize that the methodology shown is simplistic and for illustration purposes only. We refer to Factor Investment Implementation and the Rationale for Alpha and Beta Separation (Mads Gosvig and Morten T. Kronborg (4), 2019) for a more comprehensive treatment of the subject.

	Factors			
Asset Classes	**Equities**	**Nominal Rates**	**Inflation**	**Other**
Listed Equity Index	100			
Government Nominal Bonds		100		
Commodities			100[7]	
Inflation-Linked Government Bonds		92	85[8]	
Listed High Yield Cooperates Bonds	100	25		
Private Equity	95			14
Infrastructure	80	50	25	24
Real Estate	45	83	39	37
Timber	34	100	50	31

Table 3: Stylized risk factor mapping for a given asset investment of 100 units of risk (volatility)

In Table 3 we show a stylized mapping for a list of asset investments each of 100 units of risk (volatility or Public Market Equivalent volatility for illiquid assets as explained below) in size.

An established listed equity index investment (e.g. MSCI World hedged to the investors base currency) could be mapped one-to-one to the Equities Factor. The actual index may not be equivalent to the Equities Factor used in the previous chapters; however, any globally diversified equity index could in principle be applied as the Equities Factor.

Similar, 'safe' nominal cash flows, such as a US or German bonds, could be mapped one-to-one to the Nominal Rates Factor. The choice of bonds may vary; the main considerations are that the bonds chosen should be regarded as global 'safe haven' assets. Less safe bonds may contain a credit related risk attributable to the Equities Factor.

Safe real cash flows, such as US and German inflation-linked bonds, could be mapped to both the Nominal Rates Factor and to the (output) Inflation Factor, i.e. 10-year BEI inflation. Note that slightly less risk is mapped to the Inflation Factor than to the Nominal Rates Factor

[7] 100 unit of risk to a basket of commodity futures.
[8] 85 unit of risk to 10Y BEI swap.

since the duration of inflation-linked bonds are higher than for nominal bonds with the same time to maturity, due to positive expected future inflation. The raw sum of the two exposures (92+85) is much higher than the initial 100 units of risk due to diversification between the two factor exposures.

A well-diversified basket of commodity futures could be mapped one-to-one with the (input) Inflation Factor.

High yield corporate bonds could be mapped along the lines of Merton (1973) where the equity loading has an option-delta interpretation reflecting the rating. We map high yield corporate bonds as an Equities Factor exposure plus a smaller (1/4) Nominal Rates Factor exposure. As mentioned above less safe sovereign debt can be mapped like high yield corporate debt with the convention that lower credit quality implies a higher Equities Factor loading.

The factor-mapping of illiquid investments is less of a given thing and even the factor-representation of illiquidity risk in the Other Factor is up for discussion. For example, consider private equity which could be mapped to the Equities Factor and the Other Factor. The exposure to the Equities Factor could be a listed equity index times an appropriate leverage adjustment (1.4 is our experience) as private equity generally is more leveraged than public equity. The exposure to the Other Factor could account for illiquidity risk or more specifically the illiquid residual also known as the ‘‘illiquid-liquid basis” in the following sense: The most important driver of the private-public equity residual might be explained by reduced investment flexibility in times of liquidity shortage. When liquidity dries up, some investors may run into cash shortage and are forced sellers. This may affect prices substantially to the down side and makes it unattractive or even impossible to sell investments during such periods. This reduced investment flexibility could be viewed as written optionality on the ability to sell illiquid assets. We therefore represent the illiquid residual under the Other Factor with a sold put-option on a liquidity related index with an appropriate principal. The principal is 75-125% of the notional depending on the level of sensitivity towards the business circle, the complexity of the deal, and undrawn commitments (all attributes that increases illiquidity in a stressed market). This representation follows Longstaff (1995, 2014) who model the (upper bound of the) illiquidity price-discount in a Black-Scholes economy as a function of the ‘‘total variance” (the product of the implied volatility squared and the illiquid lock-up period).

In our factor approach real assets, like infrastructure, real estate and timber, are a balanced mix of the four factors. This is a version of Public Market Equivalent (PME) representation of illiquid assets. We decompose assets into nominal cashflow potentially with some form of inflation indexation (like inflation-linked bonds), an equity like participation related to e.g. occupation rate for real estate, and then obviously illiquidity. We consider the risk of the assets (in Table 3 the volatility) as the risk on the PME representation of the asset.

As illustrated in Table 3 all three types of investments are in our view rather balanced factor investments. Real estate and timber have more Nominal Rates Factor than Equities Factors exposure. The opposite is the case for infrastructure investments. All real asset classes have a slightly overweight to the Inflation Factor and the Other Factor compared to *Balanced Beta+*.

6. Conclusion

At ATP we have built a factor approach to investing. The purpose of this paper is to present the back bone of our factor approach and quantify the numerous choices made in the process of development. We describe our four main investable factors, consider their historical return histories, co-variation properties and macroeconomic biases. The *Balanced Beta+* portfolio, with loadings to the four factors, presented in this paper, represent our total return portfolio and acts as our risk-free exposure around which the organization can take initiative.

This systematic way of operating does not by construction lead to better portfolios, but it makes portfolio decision making transparent by recognizing the major drives of the assets' risk and returns. Our rather high exposures to alternatives such as break-even-inflation, commodity futures, private equity, infrastructure, real estate, timber, and several liquid systematic factor trading strategies, may make us an atypical institutional asset owner. All these exposures fit perfectly into our risk based factor approach as the preferred way to obtain diversification, robustness and consequently, by use of leverage, to a low risk portfolio, the best path to deliver the required rate of return.

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