Dynamic Management of Portfolio Risk

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Abstract

In ATP, an old age supplementary pension scheme, pension benefits are guaranteed nominal annuities. The ultimate goal of ATP's investment portfolio is to achieve a return high enough to increase the nominal pension benefits. High returns on the investment portfolio translates into increases of the guaranteed annuities through profit sharing (pension bonus) so that the value of pension benefits in real terms can be maintained. Achieving high returns is subject to keeping net assets above a certain threshold. This is managed through a probability-based risk tolerance mechanism to control the overall risk. The risk tolerance is set by the board of directors and cover multiple types of risks where market risk related to investments and longevity risk related to the pension provisions are the dominant types. The risk tolerance translates into a risk cap for market risk in the investment portfolio which depends on prior returns and thus gives rise to a requirement for dynamic management of portfolio risk. Within this framework, the two main questions are how much market risk is needed to achieve our investment return goal and how to implement a dynamic management of market risk to stay within ATP's risk tolerance. This paper outlines our pension driven investment framework: the separation of liability hedging of the guaranteed pension benefits and the investment portfolio; the top-down management of risk; and the formation of a dynamic market risk target for the investment portfolio based on quantitative analyses and qualitative judgement. We conclude that our way of managing and taking on market risk balances being prudent and achieving our investment goals at the same time.

This paper is number two in a series of four papers on ATP's investment management. The other three papers are: A Balanced Factor Approach to Investing (Lorenzen, Gosvig, & Kronborg, 2019), Local Factor Dynamics (Mads Gosvig and Morten T. Kronborg (3), 2019), and Factor Investment Implementation and the Rationale for Alpha and Beta Separation (Mads Gosvig and Morten T. Kronborg (4), 2019).

Keywords: Portfolio separation, liability hedging, CPPI, risk budget, risk control, profit sharing, defined contribution, pension guarantee.

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1. Introduction

For a pension fund with obligations to pay out pension benefits far out in the future, the conventional wisdom is that the investments should be 'long-term' and that the pension savers should be able to benefit from this 'long-termism'. Some proponents of long-termism argue that pension savers should not worry about short-term losses as history shows that short-term losses in a well-diversified portfolio turn into gains in the long run. Opponents would say that 'hope is not a strategy' and design processes to effectively manage potential outcomes and meet return goals.

At ATP we have developed our approach to being long-term for almost 20 years. We protect our guaranteed nominal pension benefits by splitting our investments into two portfolios: a liability hedging portfolio and a total return investment portfolio. The liability hedging portfolio generates (with a very high level of safety) the needed cashflows to pay our pension obligations. Our investment portfolio seeks to achieve our investment return goal and is invested and controlled by a risk budget which depends on the size of the surplus (assets minus present value of obligations). The aim is to keep the probability of a negative surplus very low at all times. Hence, we adjust our investment risk dynamically over time to keep the probability of a negative surplus low. This way of operating is known from academia as a Constant Proportion Portfolio Insurance (CPPI) strategy.

In this paper we present our dynamic management of portfolio risk which is a practical version of a CPPI strategy, incorporating both short- and long-term risk. Short-term risk is the risk of a negative surplus if a bad event happens right away. Long-term risk is the risk of not achieving our investment return goal (over a 'pension' long horizon). We demonstrate by use of our asset-liability-risk-management (ALRM) model that this way of operating balances being prudent and achieving our investment goals at the same time.

The outline of the paper is: In Section 2 we describe our pension fund setup which is essential to understand the framework within which we operate. In Section 3 we go into details about how we have chosen to model our pension framework in our ALRM-model. Finally, in Section 4, the main body of the paper, we present multiple analyses of our way of managing market risk. This is done by use of both historical and model-based simulated market returns, and by focusing on statistics on achieving investment goals and avoiding bad states.

2. ATP's pension fund governance

Before presenting our dynamic management of total risk (Section 4) there are several important aspects of the ATP pension product and its governance which are relevant for the design of the market risk strategy. These are presented in this section. Readers familiar with the ATP pension product may skip this section and jump to Section 3. Many of the aspects are relevant for defined contribution pension schemes with collective profit sharing in general.

The pension product

The ATP pension product is a contribution-based lifelong annuity made up of accumulated guaranteed nominal cash flows originating from pension contributions and ad hoc pension increases over time. The current pension product at ATP is described in (Jarner & Preisel, 2016).

For every contribution made by an ATP member, 80% of the contribution is converted into a lifelong annuity with guaranteed nominal cashflows starting upon the retirement day. The size of the annuity cashflows per contributed amount is set annually based on an interest rate which is calculated from the prevailing market interest rate term structure and a mortality surface. The present value of the annuities is called *the (actuarial) Reserve (R)*.

The other part of the contribution, the residual 20%, is transferred to a collective pool of funds known as the *Surplus (S)*. The aim of the collective fund is to grow through high investment returns, thereby making room for ad hoc pension increases as a profit-sharing mechanism. The profit sharing increases the guaranteed nominal pension benefits and decreases the Surplus and is known as *pension bonus*.

Below we describe these three critical terms in more detail; the Reserve (R), the Surplus (S) and pension bonus. We also describe how we invest through two separate funds, what risks ATP faces and how we measure them, and how our risk budget (limit) is derived from the value of the Surplus.

The Reserve (R) - market consistent value of the annuities

From an accounting perspective, the guaranteed annuities are a liability to ATP. ATP is a fully funded pension scheme in the sense that the present value of the liabilities must be less than the *total value of the assets (A)* at any point in time. ATP's assets and their future returns are the sole source of the pension benefits. Future contributions may not be relied on.

The most transparent way to assess whether assets are sufficient to cover liabilities now and in the future is through mark-to-market valuation of both assets and liabilities. Market prices reflect the market's expected present value of future asset returns. When evaluating the present value of the guaranteed annuities, we use what is known as *market consistent valuation* which is an actuarial value of the future guaranteed pension benefits. The term *market consistent* arise from the fact that we use market valuation based on 'default free' interest rate instruments[4]. When discounting liabilities with respect to mortality, we use our best estimate on expected survival probabilities which includes a trend towards lower expected future mortality. The *market consistent value of the annuities* is called *the (actuarial) Reserve (R)*. A detailed presentation of the market consistent value of the liabilities can be found in (Jarner & Preisel, 2016).

Market-based discounting of liabilities brings assets and liabilities on the same page. The key driver is what markets expect. This is transparent and keeps us aligned with reality.

Surplus (S)

At ATP the key risk metric is how large *net assets* (i.e. assets less the present value of the guaranteed annuities) are and not the assets themselves. This is a consequence of the mark-to-market principle, as the present value of the guaranteed annuities changes when market rates change. We refer to net assets as the Surplus (S). We get

$$S = A - R \quad (1).$$

Portfolio separation (two portfolios)

From a risk management perspective, the natural origo of market risk is not a cash position, but a portfolio protecting the Surplus. This leads us to one of ATP's fundamental investment beliefs; that portfolio separation with a *liability hedging portfolio* aiming at minimizing the impact from marked induced changes in the market value of the pension provisions and an *investment portfolio* aiming at producing high returns is the best way to

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[4] For ATP Danish and German government bonds and interest rate swaps are used up to 40 years maturity. For longer maturities a fixed rate of 3 percent is applied.

deliver pensions with a lower bound for pension benefits. Portfolio separation makes the total risk more manageable while improving transparency. This is in line with the theory for a Constant Relative Risk Aversion (CRRA) utility investor with infinite marginal utility for wealth near a lower acceptable bound B. It can be shown that the optimal solution consists of investing wealth in two funds: a part of the initial wealth is used to hedge the lower bound B, and the remaining part of the initial wealth is invested equivalent to the non-restricted problem (obtained by setting $B = 0$, known as the Merton problem (see (Merton, 1971))). This is known as Constant Proportion Portfolio Insurance (CPPI) (see (Black & Perold, 1992)).

The liability hedging portfolio

In theory, the liability hedging portfolio would be a portfolio consisting of a collection of nominal zero-coupon bonds with cashflows exactly matching the expected annuity cashflows. In practice, this is not possible, and we aim for a portfolio that minimizes the impact on the Surplus from market induced changes to the discounted value of the guaranteed pension benefits, i.e. changes in the value of the Reserve due to shifts in the interest rate term structure. This is known as duration matching. The liability hedging portfolio is predominantly invested in Danish and German government bonds and interest rate swaps. In order to eliminate risk related to changes to the curvature of the discount yield curve we use multiple points on the term structure. Recalling that 80% of the initial pension contribution is converted into lifelong annuities with guaranteed nominal cashflows, in terms of capital, the liability hedging portfolio is the major portfolio at ATP.

The investment portfolio

The investment portfolio is a nominal total return portfolio aiming at achieving our investment return goal (detailed further below). The asset allocation mix is based on investable factors as described in (Lorenzen, Gosvig, & Kronborg, 2019). The cash 'belonging' to the investment portfolio equals the value of the Surplus. However, leverage[5] is used to obtain the desired risk for the investment portfolio. This is done through an internal loan from the liability hedging portfolio (which has free cash available due to the use of interest rate swaps for parts of the duration hedging of the guaranteed annuity liabilities) and by use of derivatives. How to dynamically adjust the risk level of the investment portfolio is the main objective of this paper.

Risks, the risk budget and the market risk budget

ATP faces multiple kinds of risks that may affect the Surplus. Most notably, *market risk* from investments. Other risks are *pension risk* in the form of longevity risk (the 'risk' that people live longer on average than expected), *hedging risk* (due to imperfect match between the liability hedging portfolio and the guaranteed pension annuities), *default risk* (risk that counterparties on derivative contracts default) and *operational risk*, all which we group into what we call *non-market risk*[6].

If the Surplus is positive, the fund is said to be solvent[7]. However, we do not only focus on being solvent today but also in the future. In fact, we aim for a high probability that the Surplus stays positive in the future. Potential scenarios are simulated, including both market returns and non-market effects, and the resulting Surplus distribution is considered. If too many

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[5] In this context the term leverage simply means investing using cash borrowed internally or using derivatives.

[6] We will not discuss liquidity risk or other kind of relevant risk for ATP in this paper.

[7] The term 'solvent' is used here even though ATP is not subject to a formal solvency requirement.

of the simulated Surplus outcomes are below a certain threshold, the risk is considered too high. The threshold depends on the level of risk aversion, the risk measure and the time horizon.

It is worthwhile to stress, that viewing the Surplus as the object to protect with high probability and not 'just' an amount of cash to invest is key to ATP's investment approach. Relieving the investments from the cash binding and allowing the investments to be risk driven gives ATP room to operate more flexibly and to invest in a much more well-diversified portfolio. Surely, it also raises the bar for being prudent as the portfolio complexity is higher.

For ATP, the maximum risk limit is when the expected decrease in the Surplus three months from now, conditional on being in the worst 1 percentile of the scenarios, equals 50% after tax[8]. This risk limit defines what we call the *risk budget*. In mathematical terms we have the condition

$$E(-\Delta S_{3M} \mid \Delta S_{3M} < x_{0.01}) \leq \frac{1}{2} S_0, \quad (2)$$

where S_0 is the value of the initial surplus, $\Delta S_{3M} = S_{3M} - S_0$ is the change in the Surplus over the next 3 months with distribution function G, and $x_\alpha = G^{-1}(\alpha) = inf\{x \epsilon \mathbb{R}: \alpha \leq G(x)\}, \alpha \epsilon [0,1]$.

The primary risk that we can control is the market risk from the investment portfolio. For a given level of non-market risks we derive a *market risk budget* from (2) which is illustrated in Figure 1[9].

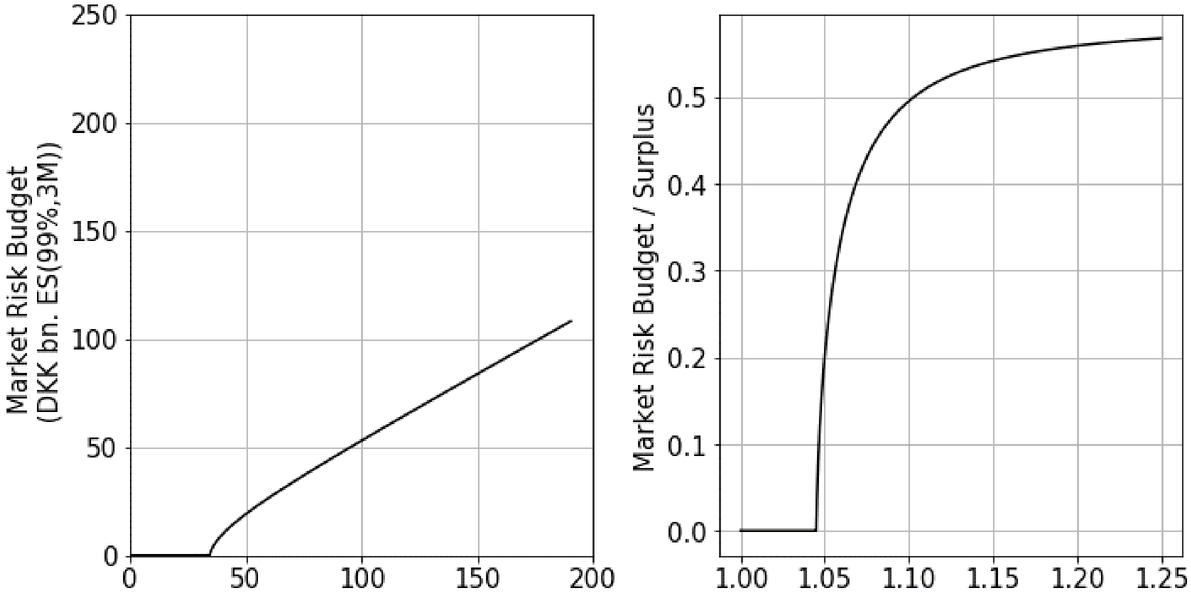

Figure 1: The market risk budget for the investment portfolio (pre 15.3% market return tax): Left: The market risk budget as a function of the Surplus given the Reserve value mid-2019. Right: The market risk budget relative to the Surplus as a function of the Funding Ratio.

The concave shape of the market risk budget is due to diversification between market risk and non-market risk which is approximately quadratic. Observe from the left plot in Figure 1 that the market risk budget is zero at Surplus values below a certain level (DKK 34.3 bn. in Figure 1). Below this level the non-market risk already exceeds the risk budget, hence there is

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[8] For Danish pension schemes tax on market returns equals 15.3%

[9] Market risk is measured as an instantaneous chock to current positions, i.e. the expected loss without any management actions over the time period. Market risk is shown before tax, i.e. the potential effect on the Surplus is 0.847 times the market risk.

no room for investments besides the liability hedging portfolio. The non-market risk varies over time with the size of the Reserve, so the absolute market risk budget in the left plot of Figure 1 changes in practice slightly with changes in the Reserve (non-market risk is approx. 2.25% times the Reserve value). However, for shorter periods the practical effect on the market risk budget is small. The right plot in Figure 1 illustrates the market risk budget as a fraction of the Surplus as a function of the Funding Ratio (defined below), and consequently, the plot stays the same for all values of the Reserve.

We discuss the effects and implementation of the dynamic market risk budget approach in Section 4.

Pension bonus – the ultimate goal of the investment portfolio

Pension bonus (the profit-sharing mechanism) is triggered when the Surplus becomes sufficiently large. Then funds are reallocated from the Surplus (S) to the Reserve (R) and the funds are used to increase the guaranteed annuities for the members. All pensions rights are increased proportionally, since the Surplus is a collective pool of funds. This insures fairness among the members. At ATP a possible pension bonus is considered annually by the board of directors[10].

More formally we define the funding ratio as

$$F := 1 + \frac{S}{R}.$$

A necessary, but not binding, condition for the board to allocate pension bonus is

$$F > \kappa, \text{ where } \kappa = 1.2.$$

For this article we allocate pension bonus on an annual basis using the bonus barrier strategy given by:

$$r^b(t_i) = \max\left(\frac{F(t_i -) - \kappa}{\kappa}, 0\right), \quad (3)$$

where $r^b(t_i)$ is the bonus rate (the rate at which pensions rights are increased) at time t_i, and $t_i -$ is the time just before a possible pension bonus. Note that at the time of a possible pension bonus the funding ratio becomes

$$F(t_i) = min(F(t_i -), \kappa),$$

i.e. a barrier strategy.

In this way, over the lifespan of the member's pension saving process, the returns on the Surplus is gradually added to the pension benefits in years when returns have been sufficiently large and the Surplus becomes sufficiently large. These increases in the pension benefits are the ultimate goal of the investment portfolio. An increase in the pension benefits translates into an increase in the liability hedging portfolio (because an increase in the pension benefits increases the sensitivity to changes in discount rates and hence increases the requirement for hedging) and a reduction in the risk budget (because the risk budget is an increasing function of the Surplus and a decreasing function of the Reserve).

In a market diffusion model disregarding non-market risk, closed form expressions are available for the statistics of the bonus mechanism and the total pension product. See (Jarner & Kronborg, 2016) for a comprehensive analysis with roots in random walk theory.

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[10] The board of directors also has another pension bonus mechanism available that allocates pension bonus to retired members only. This bonus mechanism is not modelled in this analysis.

Cash

The implementation of portfolio separation into a liability hedging portfolio and an investment portfolio is a separation aiming at controlling risk. The two portfolios share the total cash pool, but for transparency purposes each portfolio has its own cash allocation. This defines the term ‘value’ of the liability hedging portfolio and the investment portfolio, respectively, where the value of the former is equal to the Reserve and the value of the latter is equal to the residually defined Surplus. Transfers of cash between the portfolios are tracked by an internal loan. This internal loan (where the smaller, in cash terms, investment portfolio borrows cash from the larger liability hedging portfolio) allows for leverage of the investment portfolio relative to its cash value.

All the mechanisms described above are illustrated in Figure 2 of Section 3. First, we need to present our modelling framework for evaluating the pension scheme model.

3. Our modelling framework

The model used in this paper is an Asset-Liability-Risk-Management (ALRM) model. The model is a simplification of the day-to-day governance of ATP (the actual models and governance used by ATP). We aim at simplicity to achieve an agile model with a primary focus on the asset side and investment management actions, while at the same time capturing the big picture of our pension setup and risk management governance.

For market returns we have two approaches:

1) Historical simulations: We use historical returns from major markets across regions and asset classes. All foreign investments are hedged to DKK. The data period covers 1974-01 to 2019-10.
    a. The investment portfolio is invested in our reference factor portfolio Balanced Beta; a well-diversified low-risk portfolio with loadings to our Equity-, Rate- and Inflation Factor (see (Lorenzen, Gosvig, & Kronborg, 2019)). When constructing the investment portfolio in each step, derivatives and not cash from the liability hedging portfolio, are used to provide leverage to the investment portfolio for simplicity; this does not alter the results.
    b. The liability hedging portfolio duration match liabilities by investing in 10-year maturity German bonds. To reach duration match, the investments in the liability hedging portfolio has a value equal to 1.5 times the Reserve, i.e. in the model the liability hedging portfolio is levered compared to the value of the Reserve. This is simply a consequence of only investing in the 10-year bond and does not alter the results materially. In real life, investments are made in multiple bonds and swaps with different maturities without applying leverage.
2) Model simulations:
    a. The (log-transformed) excess returns of the investment portfolio is modelled by a GARCH(1,1)-NIG process with a (log) return-to-risk ratio of 0.38 representing our expected return after operating costs[11] going forward. The model is estimated using data (historical returns) from our reference factor

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[11] Recall that risk is measured as the 3 months 1-percentile expected short fall and not volatility. The return-to-risk ratio corresponds to a Sharpe Ratio of approximative 0.55 before operating costs.

portfolio Balanced Beta. However, we believe that historical returns may be an over optimistic representation of future returns, and consequently we have adjusted the drift parameter of the model to represent our internal assumption about expected return per risk. For a description of our market return model see the Appendix.

b. The excess returns of the liability hedging portfolio yields a constant = 1.5%[12], i.e. the slope of the terms structure is static.
c. The short rate is a constant = 0%[13].
d. 100000 simulation of 50 years in steps of months are generated.

Non-market risk

In the real-life risk model used by ATP, the ratio of non-market risk to the value of the Reserve is close to constant. Hence, we model non-market risk by a constant fraction (2.25%) of the Reserve. Non-market risk feeds into the calculation of the market risk budget, but in this analysis non-market risk is not realized, i.e. the hedging of the liabilities is a perfect fit and mortality realizes just as expected.

Market risk

As explained in Section 2 the market risk is quoted in terms of the 3 months 1-percentile expected short fall of the investment portfolio (ES(99%,3M)). We calculate ES(99%,3M) assuming a log-normal distribution. We assume the investment returns to be log-normal distributed and use, at any point in time, 12-years[14] of past return data for all positions in the investment portfolio to estimate the distribution. To reflect common risk-management practice we adjust the drift of the distribution to zero such that the median return equals zero. The risk is for current positions, i.e. the expected loss without any management actions over the time period.

In our analysis we use this approach on both historic and GARCH(1,1)-NIG simulated returns. Both return series fail the log-normal hypothesis due to non-normal values of skewness and kurtosis and due to autocorrelation of excess returns. However, the log-normal distribution is the industry standard and the model used by ATP. In Section 4 we show that *measuring* market risk using the log-normal assumption for current positions and without assuming any management actions and *managing* the risk within the risk budget effectively strikes the balances between being prudent and achieving our goals in a reasonable way.

Illiquid assets

ATP's investments include liquid as well as illiquid assets such as private equity, real estate and infrastructure. The value of illiquid investments is in practice less responsive to changes to

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[12] The assumption of a constant (positive) yield for the liability hedging portfolio (i.e. a constant sloping term structure) is a simplification. However, over longer time periods excess returns for duration risk is believed to be constant. For the analysis made in this paper, the main effect is that the value of the Reserve will have a constant positive drift. The level of this constant drift is of course debatable.

[13] The assumption of a constant zero short rate in the simulations is obviously not realistic. However, since both the liability hedging portfolio and the investment portfolio benefits from (higher) short rates, it is the magnitude of excess returns for the investment portfolio that predominantly determines the ability to create pension bonus. As we believe excess returns generally to be unrelated to the level of the funding rate, we believe the assumption of constant short rate is reasonable over longer time periods.

[14] A 12-year data window reflects the fact that currently ATP does not want the 2008-2009 financial crisis to be absent from the data estimation set.

comparable liquid markets and hence significantly less volatile than liquid investments. In the real-life risk model used by ATP, illiquid investments are modelled using a public market equivalent representation as outlined in (Lorenzen, Gosvig, & Kronborg, 2019). For the purpose of this analysis, we model all investments as liquid investments.

Pension flows

We consider neither pension contributions nor benefits. The interpretation is that the pension fund is currently in stationarity with pension outflows matching contribution inflows. This is close to the situation for ATP in real life.

Pension bonuses are paid in accordance with (3) on an annual basis.

Frequency of management actions

We consider monthly returns, and consequently management actions (re-calibration of absolute and relative market risk) are performed on a monthly basis.

An illustrative example of how it works

In Figure 2 we illustrate our model and all the mechanisms described in Section 2 and 3. We use historical market simulations and consider the period 1995-2004, a period which includes the burst of the tech bubble and a fall in interest rates by just below 400 bp[15]. We start the balance sheet in January 1995 with values equal to the mid-2019 value of ATP's total assets and Reserve. The interpretation is; what would happen if for the next 10 years the marked returns from the beginning of 1995 to the end of 2004 materialize (again). We emphasize that this is an illustration of the mechanisms at work without regards to whether the portfolio is manageable in such size in real life. The (more than) tripling of total assets in the ten years period is driven partly by higher level of interest rates (and thus compounding of the increase in the Reserve), partly by the fall in interest rate by almost 400 bp, thereby boosting the hedging portfolio and the bonds and equities in the investment portfolio. Hence, the tripling is by no mean unrealistic, but will most likely not repeat itself given current interest rate levels. Further this cannot be compared to how ATP actually fared in real life during the same period, as neither the starting point nor the strategy applied then are comparable.

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[15] For Germany 10Y bonds 389 bps (from 7.44% to 3.55%) and for US 10Y bonds 346 bps (from 7.60% to 4.14%).

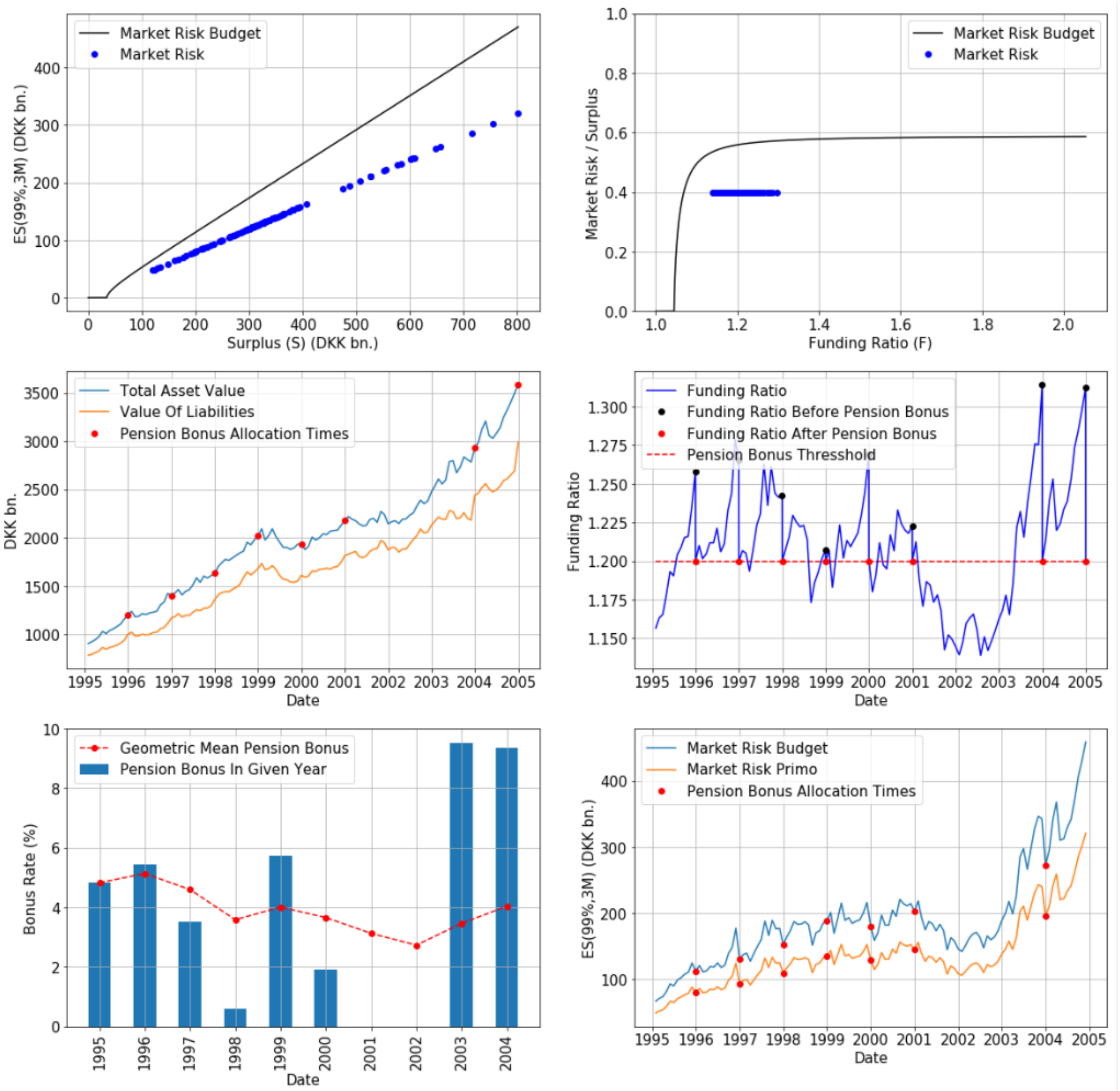

Figure 2: Hypothetical simulation of ATP's pension product and investments using market returns from January 1995 to December 2004. Top Left: The Market Risk Budget (pre simulations) and the Market Risk Strategy against the Surplus. Top Right: The Market Risk Budget and the Market Risk Strategy relative to the Surplus, both against the funding ratio. Middle Left: The Liability Hedging Portfolio and Total Assets. The value of the Surplus (the cash value of the Investment Portfolio) equals the gap between the two. Middle Right: The funding ratio with dots indicating the funding value just before and after a possible pension bonus allocation. Pension bonus is allocated as a barrier strategy with a barrier value of 1.2. Bottom Left: Size of a possible pension bonus in a given year and the cumulative geometric mean pension bonus. Bottom Right: The Market Risk Budget and the Market Risk deployed according to the Market Risk Strategy by the Investment Portfolio.

In this illustration we apply a market risk strategy which rebalances the market risk-to-Surplus ratio at a constant level of 0.40, subject to being within the market risk budget. For the period of consideration, the Surplus stays fairly high at all times and, consequently, the market risk does in fact not exceed the market risk budget during the period. The strategy is illustrated in the top right plot of Figure 2. In absolute terms, as seen from the top left plot of Figure 2, this means that the DKK market risk is increased as the surplus increases. In the middle left plot of Figure 2, the value of the liability hedging portfolio and total assets are illustrated. The gap between the two equals by definition the value of the Surplus. In the middle right plot of Figure 2, the pension bonus mechanism is illustrated. Whenever the funding ratio exceeds 1.2 at year-end, bonus is allocated such that the funding ratio drops to 1.2. The size of the yearly bonuses and the geometric mean pension bonus over the period (roughly 4%) is illustrated in the bottom left plot of Figure 2. In the bottom right plot of Figure 2 we see that the market risk budget, and consequently for this investment strategy, the market risk, decreases after a possible pension bonus allocation.

4. Balancing market risk between being prudent and achieving pension goals

At the highest level, the goals of the investment management at ATP is to keep net assets above a certain threshold and to strive towards increasing pension benefits enough to maintain their value in real terms through profit sharing (pension bonus). The latter goal is subject to not compromising the former goal.

As we have explained, the former goal is to a first order handled by a dedicated liability hedging portfolio (only resulting in a small amount of tracking error risk) and actively managing the risk of the Surplus becoming too small.

The latter goal is obtained if the geometric mean pension bonus is above the realized inflation. For this to be expected a certain amount of market risk is needed in the investment portfolio which we first consider heuristically in Subsection 1) below. In this analysis we focus on strategies which can achieve a certain fixed geometric mean pension bonus level. The analysis does not model inflation explicitly, and thus we constrain the analysis. We apply an inflation target equal to 1.75% as this is the current expected long-term level of Danish inflation.

In Subsection 2) we present our market risk strategy based on quantitative analysis and qualitative judgement. We analyze the market risk strategy from two perspectives. In Subsection 3) we put the strategy into a historical perspective (and an expectation adjusted historical perspective) and consider the market risk dynamics during four selected crises and compare the fund to two classic total return portfolios. In this way we show that our market risk strategy is durable for controlling risk, and at the same time the returns for the total fund keeps up with alternative approaches, despite offering a guaranteed pension cashflow. In Subsection 4) we use model simulated market returns and consider both pension bonus and ‘bad state’ statistics. By doing so, we hope to show that the compromise between being prudent and being true to our pension goals are well balanced.

1) Return requirement and market risk needed to achieve goal – intuition from heuristic calculations

Before presenting and analyzing the market risk strategy, it is useful to present some heuristics about what level of return are required from the investment portfolio to achieve a certain pension bonus level, and consequently what *lower* level of risk is necessary subject to assumed return-to-risk ratios.

Since the value of the Surplus is inferior to the value of the Reserve by at least a factor 1:5 (as a higher Surplus would be allocated as pension bonus), the investment portfolio must deliver a substantial return to achieve its goal of delivering meaningful pension bonuses. At first glance one could think that the investment portfolio should simply generate an 8.75% annual return (5 times 1.75%) to generate 1.75% pension bonuses. However, this approximation is inadequate for several reasons. First, 1/6 of the returns need to be kept within the investment portfolio in order to generate sufficient returns in the future (i.e. the barrier strategy defined in (3)). Second, we need to account for variations in returns, that is; after losses the Surplus is inferior to the Reserve by a factor larger than 1:5 and investment returns need to ‘fill up’ the Surplus to reach the bonus barrier again. Third, all the mechanisms described in

Section 2 complicate the interactions between the key metrics. For a complete evaluation we need our ALRM-model described in Section 3 and evaluated in the subsequently subsections, but first we address the required return and market risk needed from a heuristic point of view since much intuition can be achieved.

Assuming that pension bonus is allocated on a yearly basis using the barrier strategy in (3), the *1-year return requirement* (RR) in excess of funding for the investment portfolio to deliver pension bonus at or above a target level becomes

$$r_{excess}^{(S)} \geq \frac{(\kappa(1+i)-1)\left(1+r^{(R)}\right)}{F-1} - 1 - r_f \equiv RR \quad (4),$$

where $r_{excess}^{(S)}$ is the expected excess return on the portfolio relative to the Surplus S, F is the beginning-of-year funding ratio, κ the bonus threshold, i the target level, $r^{(R)}$ the growth rate of the Reserve, and r_f the funding rate over the period. Setting $i = 1.75\%$, $r^{(R)} = 1.5\%$, $r_f = 0\%$ and $F = \kappa = 1.2$ we get

$$RR = 12.16\%,$$

after costs, tax and other expenses including potential changes to expected pension benefits due to unexpected changes in expected longevity. Note that we assume the target level to be 1.75% and the beginning-of-year funding rate to be at the bonus threshold (its maximum value). This return requirement has the interpretation that if satisfied *every single year*, the geometric mean pension bonus will be 1.75%. In Figure 3 the risk needed, given that the return requirement is fulfilled every single year, is illustrated as a function of the assumed return-to-risk ratio. We see that for an expected return-to-risk ratio of 0.38, the needed risk is 37.8% of the Surplus before taking into account investment tax.

However, due to compounding of variating returns, the *annual expected-return requirement* (ERR) should be even higher to reach the target over multiple periods. This is known as the *variation drag*. When evaluating our market risk strategy in Subsection 3) and 4) the effect of variations in and compounding of returns is naturally considered as a part of our ALRM-model. For now, note that

$$ERR > RR.$$

To evaluate how much larger ERR should be compared to RR, and how much more risk is needed to achieve the investment goals, we need to run simulations using the modelling framework of our ALRM-model. Just to get an idea of the magnitude, remember that from the theory of continuous compounded returns the variance drag may be approximated by 0.5 times the volatility of the Surplus. This is just heuristic calculations disregarding multiple other important aspects, most importantly the bonus mechanism (which also contributes to Surplus volatility), our non-linear market risk budget (see Figure 1) and market returns being non-normal distributed. In practice, the Surplus distribution will be asymmetric, left skewed and heavy tailed. All effects that raise the bar for the true expected-return requirement for the investment portfolio. In addition, we put aside money to cover costs. Based on the above, our goal of 1.75% pension bonuses seems to require an investment strategy which combines a significant level of risk relative to the Surplus with a high return-to-risk ratio. Something we illustrate in Subsection 4) using our ALRM-model.

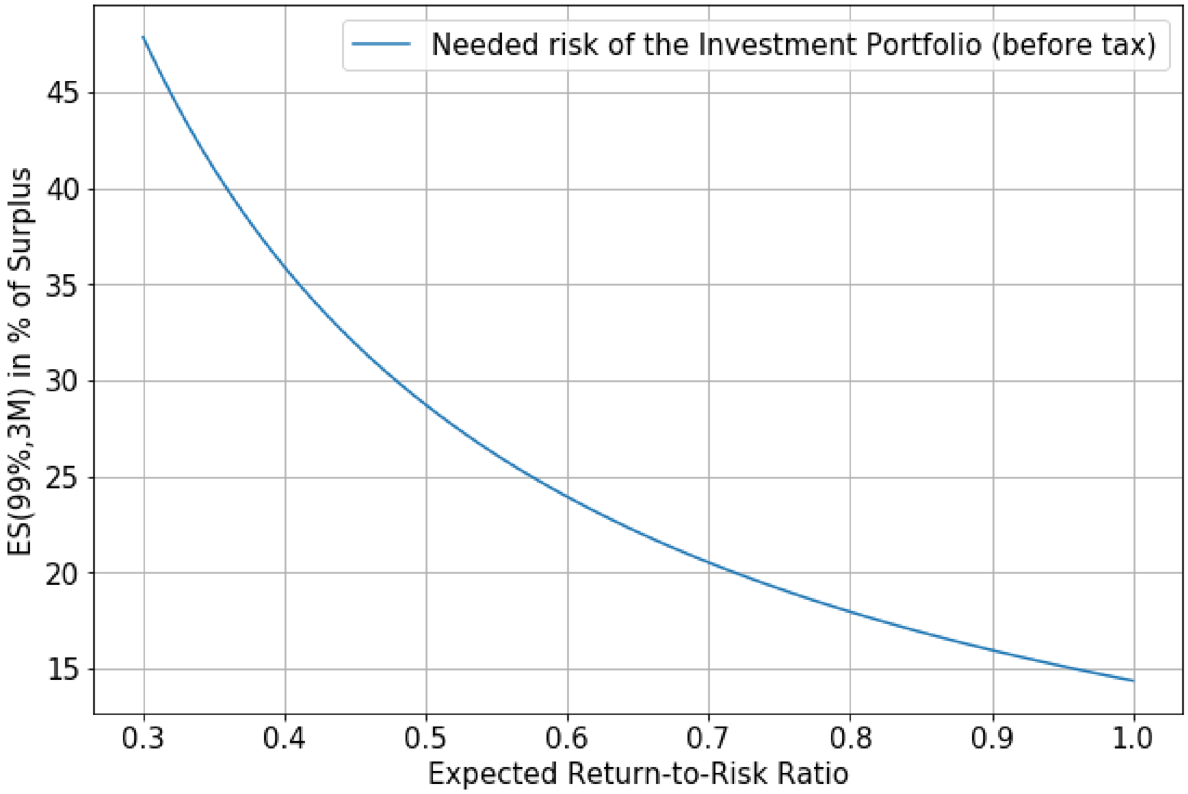

Figure 3: Blue line: Relationship between risk needed (given RR (given by (4)) is fulfilled every single year) and assumed return-to-risk ratio of the investment portfolio. Parameters are $i = 1.75\%$, $r^{(R)} = 1.5\%$, $r_f = 0\%$ and $F = \kappa = 1.2$.

How can we reach such a high expected return requirement?

The risk allocation and portfolio separation principles at play in the ATP framework imply significant differences compared to the classic cash allocation principle. In the hypothetical classic cash-allocation based investment model, the liability hedge would consist of bonds (or cash covered interest rate swaps) and the investment portfolio would invest the remaining cash. However, even if the investment portfolio was fully invested in risky assets such as equities, this investment model would not be able to generate a return high enough.

To generate higher returns, we apply diversification and leverage. The term leverage simply means investing using cash borrowed internally or using derivatives. As mentioned, an internal loan from the hedging portfolio provides funding of a levered investment portfolio, so the risk level of the investment portfolio can be set higher than reachable through traditional investment in risky assets. By relieving ourselves of the cash restriction and replacing it with a risk restriction, higher (expected) returns can be achieved.

Diversification is the other main source of high returns. We construct the portfolio with the aim to achieve a high return-to-risk ratio by focusing on balancing up investments with a high level of mutual diversification. Diversification reduces risk and increases the return-to-risk ratio as evidenced by our low risk, high expected return-to-risk ratio portfolio called Balanced Beta described in our companion paper (Lorenzen, Gosvig, & Kronborg, 2019).

2) The market risk strategy

The issue of 'appropriate amount of risk' can be dealt with from two sides: Demand for risk (to achieve the return requirement) and supply of risk (to comply with the risk budget).

From the demand side, we have just explained how the required level of risk depends on the expected return-to-risk ratio of the investment portfolio. In our asset allocation model (used below) market returns for the investment portfolio are simulated from a GARCH(1,1)-NIG model (see appendix) where the simulated return-to-risk ratio is 0.38, representing our expected return going forward minus operating costs.

As illustrated below we apply a market risk strategy taking risk relative to the Surplus when the funding ratio is high (‘normal times’), but not when the funding ratio is low (‘in the aftermath of severe losses’), thereby deliberately violating the market risk budget. By violating the risk budget, it seems like we suddenly, after large losses and subsequently a significantly smaller Surplus, do not believe in our risk management setup anymore. This is not the case. We merely prioritize other considerations in times when risks not represented in our risk model may be more pronounced. More precisely we try to balance short-term risk (present in the day-to-day risk management model) with long-term risk (not present in the day-to-day risk management setup). Long-term risk is the risk of not having risk capacity in the future and thereby not being able to reach the investment goals evaluated over a (pension) long horizon. We refer to this as absorption risk since we risk ending up in a bad state (low funding ratio) where there is no room for market risk, and consequently we will be stuck in that bad state. We have taken a practical approach to dealing with absorption risk and have chosen to put a floor on active risk reduction. If the Surplus becomes very little, we do not believe that the best solution for the pension savers is to hold zero or very few risky investments in the investment portfolio. Another way of looking at this is to focus on total assets; clearly zero investment risk on total assets (i.e. no expected investment returns over and above the guarantee) is not optimal in the long run. We are aware that this implies an enlarged risk of not being able to pay out the guaranteed pensions in the future. Consequently, we measure and illustrate the risk of being under-funded. The approach is termed ‘practical’ because it has little support in academia.

The design of our market risk strategy is a hybrid between risk being a constant fraction of the Surplus and a minimum level of risk specified by a fraction of the Reserve. At the same time the risk must not exceed the market risk budget (MRB), unless the lower risk bound is activated. We define our 40% Market Risk Strategy as (40%-MRS):

- 40%-MRS: max(min(MRB, 40% ES(99%,3M) · Surplus), 2.5% · Reserve) (5)

The 40%-MRS is illustrated in Figure 4. In Subsection 4) we also illustrate a 35%-MRS, 45%-MRS and 50%-MRS for comparison.

Remark: Even though the strategy changes the risk level over time it is not a market timing strategy. Our belief is that market-implied risk aversion *may* be predictable. However precise predictions of timing of reward for risk is highly unreliable – and certainly using ATP’s net assets as the predictor for risk aversion does not seem reasonable. Hence, the strategy is not a reflection of our timing ability.

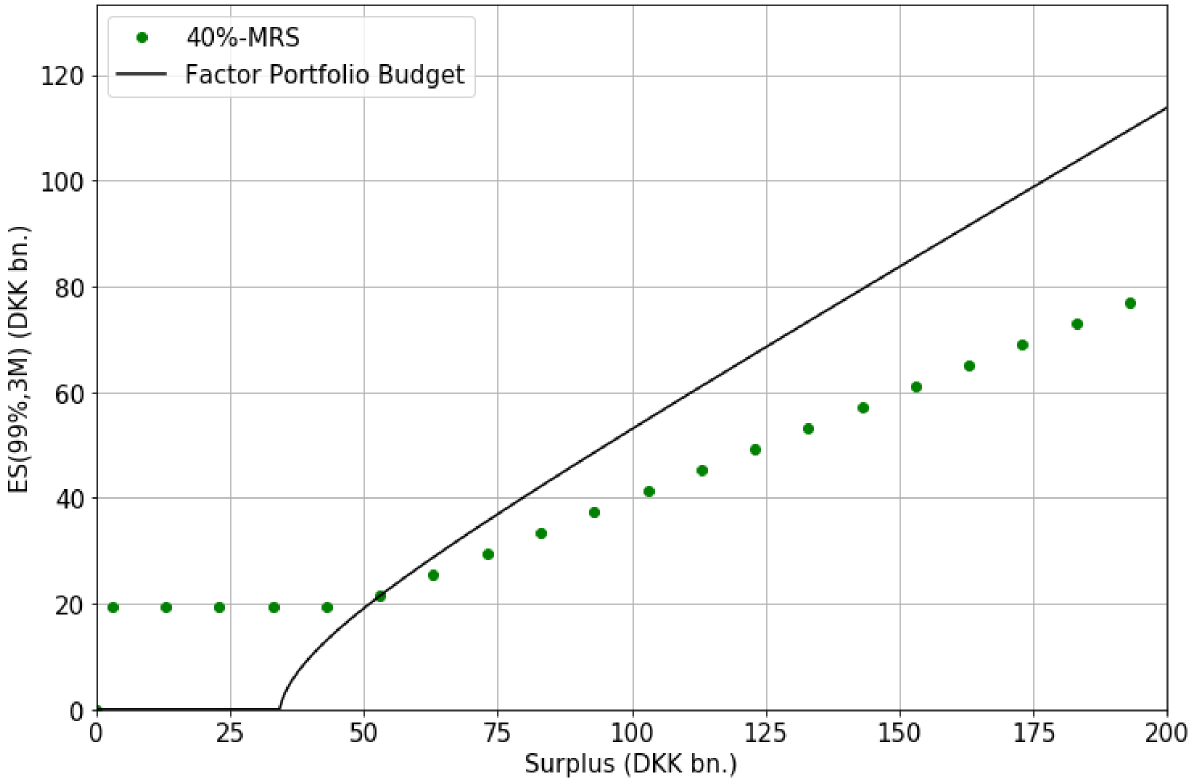

Figure 4: The 40%-MRS given a constant Reserve value as of mid-2019.

Remark: De-risking the risk after Surplus losses is a natural consequence of our desire to protect the guaranteed pension benefits. It is the natural decision given constant relative risk aversion (CRRA) preferences applied to the Surplus (see (Black & Perold, 1992)). On the other side, scaling up risk after Surplus gains is also natural given CRRA-preferences, but more importantly, it is a necessary thing to do in order to achieve the investment goals. If not done, in the long run, the average risk over time will simply be too low to generate sufficiently large returns.

3) Historic perspective

We now put the strategy to a test using historical market returns covering 1974-01 to 2019-10. The market returns used are described in 1) of Section 3 and cover our three factor portfolio Balanced Beta (see (Lorenzen, Gosvig, & Kronborg, 2019)). We also consider adjusted historical market returns. That is, we adjust historical market returns for (perceived) windfall gains (partly due to falling real rates over the time period) by simple subtracting a constant, relative to the risk, for all return series. This is done such that:

Adjusted historical returns

1) The Equity and Rate Factor are both adjusted to have a return-to-risk ratio of 0.27 (down from 0.35 and 0.45, respectively)
2) For the Rate Factor, both US and German rates have been adjusted to have the same return-to-risk ratio of 0.25 (down from 0.27 and 0.54, respectively)
3) We exclude break-even-inflation (BEI) swaps from the Inflation Factor, leaving us with commodities only, which are then adjusted to have a return-to-risk ratio of 0.15 (down from 0.22 for commodities)

4) The total return-to-risk ratio for Balanced Beta becomes 0.42 (down from 0.61)
5) The short rate (funding rate) is constant zero

In Figure 5 we plot the evolution of the pension fund's total assets (i.e. the value of the hedging portfolio plus the value of the investment portfolio when using our 40%-MRS on the investment portfolio) alongside the evolution of total assets using two alternative classic total return strategies: a cash allocated 60 percent global stocks and 40 percent global bonds portfolio and a risk balanced portfolio represented by our three factor Balanced Beta portfolio (both hedged to DKK). Both portfolios invest the pension fund's total assets and are adjusted over time to have 10% ex-ante volatility at any point in time[16]. From a historical perspective (left plot of Figure 5), our pension product with portfolio separation would have outperformed the two other industry standard approaches to fund management. This is to some extend due to falling yields over the period, boosting the performance of the liability hedging portfolio. However, if we adjust historical returns for windfall gains, as described above, our approach keeps up with the alternatives (right plot of Figure 5). This is rather impressive since the pension product offers a lower bound for performance due to the guaranteed pension cashflows. The geometric annual pension bonus received on top of the initial pension guarantee is, for reference, 2.34% and 1.50% for the historical and adjusted historical case, respectively. To be fair, as seen in Figure 6, the ex-ante risk (volatility) of our pension products total assets is slightly above the two other alternatives for most of the time. In total we conclude based on the historical analysis that a guarantee combined with a little more ex-ante risk most of the time has been a compelling product.

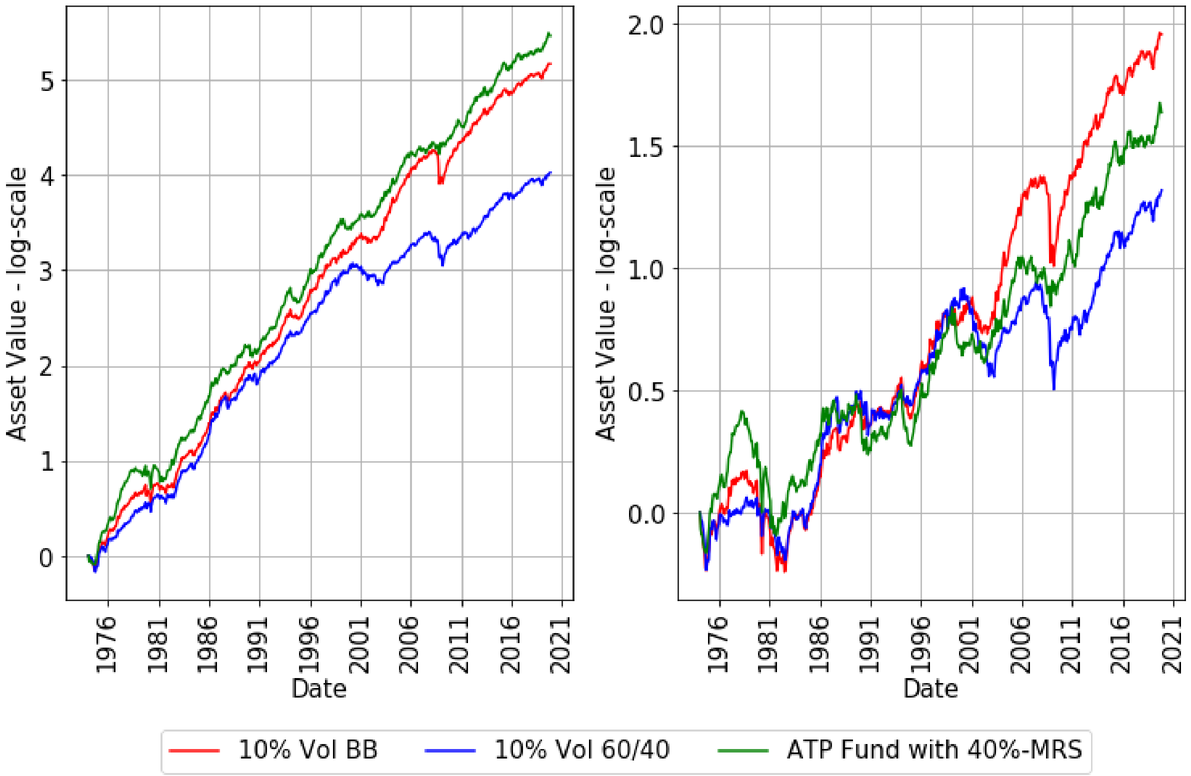

Figure 5: Cumulated returns using historic market returns (left) and adjusted historic market returns (right).

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[16] Again, using a rolling 12-year backward looking data window for volatility estimation

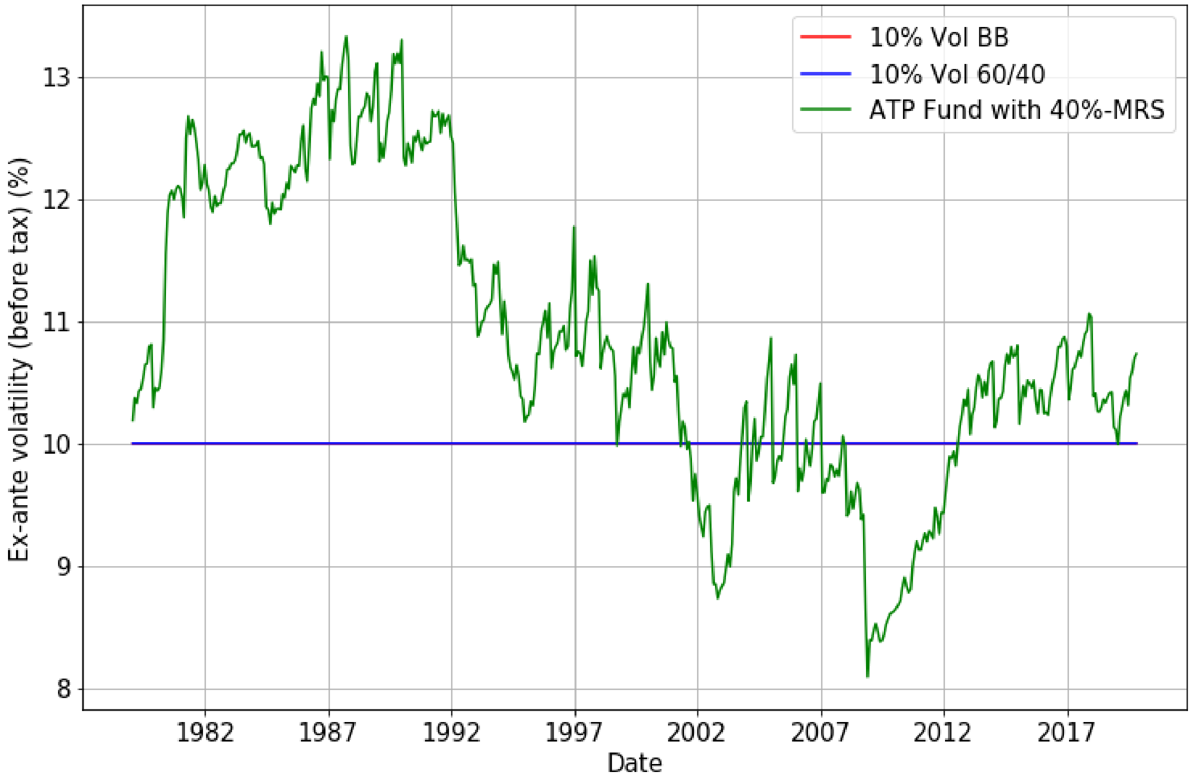

Figure 6: The ex-ante volatility (using 12-years of past returns) given the investment position at any point in time.

In Figure 7 we illustrate the historical simulated funding ratio from 1974 to 2019-10, starting with pension balance sheet values equal to the mid-2019 value of ATP's total assets and Reserve. The funding ratio increases (decreases) if and only if the investment portfolio gains higher (lower) returns than the liability hedging portfolio. The drawdowns for the funding ratio indicate periods of time where bonus is not allocated and is, in contrast to simply focusing on losses for the investment portfolio, the drawdown metric of relevance for ATP. From Figure 7 we see four larger drawdowns: The oil crisis of the 70's, the global recession of the early 80's, the dot-com equity burst in 2001 and the global financial crisis in 2008-2009. We illustrate our 40%-MRS during each of those four drawdowns in Figure 8 given pre-crisis pension balance sheet values equal to the mid-2019 value of ATP's total assets and Reserve. The interpretation is: what would happen if one of the four crises started tomorrow? We see that the drawdown during the 80's global recession is the longest, taking about four years to recover to the pre-crisis level. In comparison, the length of the drawdown is about two years for the oil crisis of the 70's and the dot-com equity burst in 2001, and three years for the 2008-2009 global financial crisis. The 2008-2009 global financial crisis differs only by having the largest drawdown. Note also that the lower bound for risk (given by 2.5% times the value of the Reserve) is only activated, thereby violating the market risk budget (see (5)), during the 80's global recession and the 2008-2009 global financial crisis. In other words, the shift from strictly short-term risk management to a minimum level of risk in order to achieve long-term investment goals are, taking the history into account, expected to happen rarely.

In conclusion, from a historical perspective, our strategy is both prudent in managing short-term risk and capable of delivering attractive long-term returns.

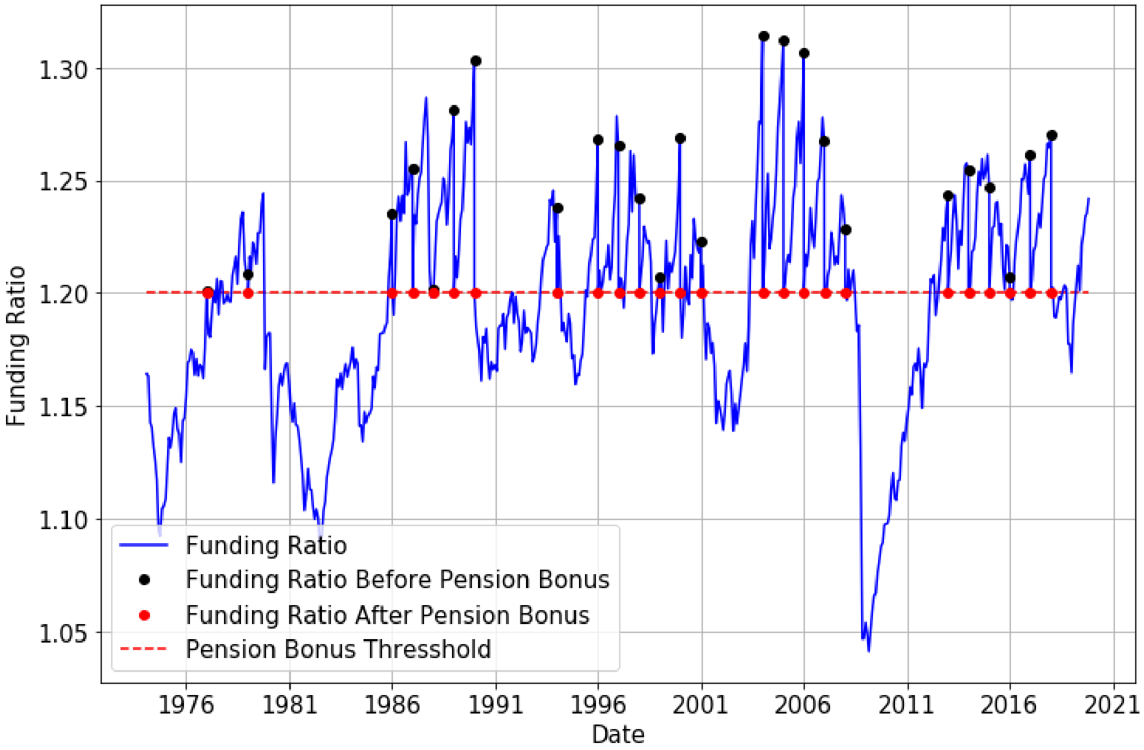

Figure 7: Historical funding ratio given our 40%- MRS, starting the simulation in 1974 with the pension balance of ATP mid-2019. The investment portfolio is fully invested in liquid assets in the analysis.

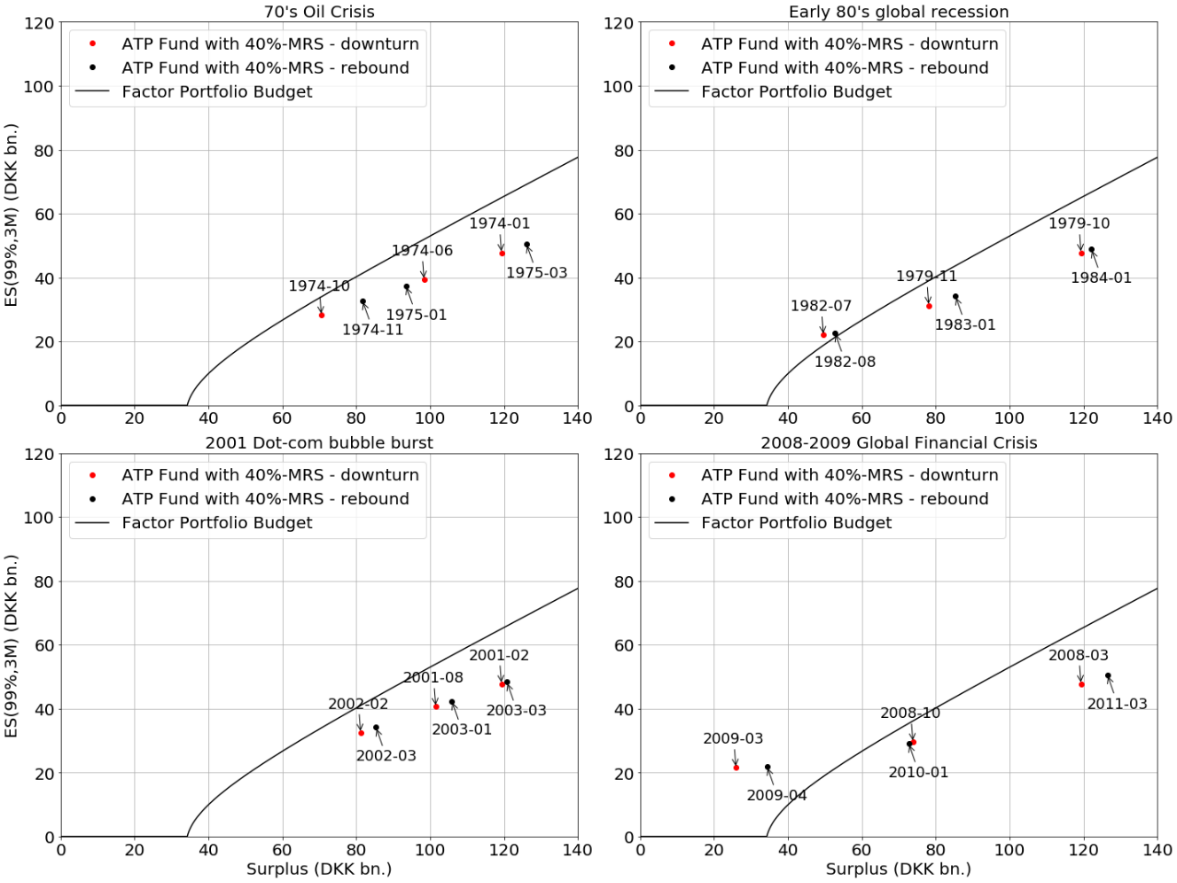

Figure 8: Illustration of our 40%- MRS during the drawdown and rebound of four selected periods. All periods start at the beginning of the drawdown with the pension balance of ATP as of mid-2019. The investment portfolio is fully invested in liquid assets in the analysis.

4) Model simulations

We now turn to our model-based simulation approach described in 2) of Section 3. Again, we focus on both the upside and the downside, and we also illustrate a 35%-MRS, 45%-MRS and 50%-MRS for comparison. Figure 9 shows the median and selected quantiles for the geometric mean bonus over time. First, we observe that the median bonus increases fast for the first couple of years (left plot). This is due to the initial values for the pension scheme simulations being equal to the mid-2019 values for ATP (funding ratio = 115.7%) which is a lower funding ratio that the expected funding ratio in stationarity. As seen the pension scheme converges to stationarity, and the median geometric mean bonus over time becomes just above 1.75% for our 40%-MRS strategy. The median geometric mean bonus, as well as the lower selected quantiles (see right plot of Figure 9), increases with the risk level. This is also illustrated in Figure 10 where we see that over the next 50-years all bonus quantiles increase with the riskiness of the strategy.

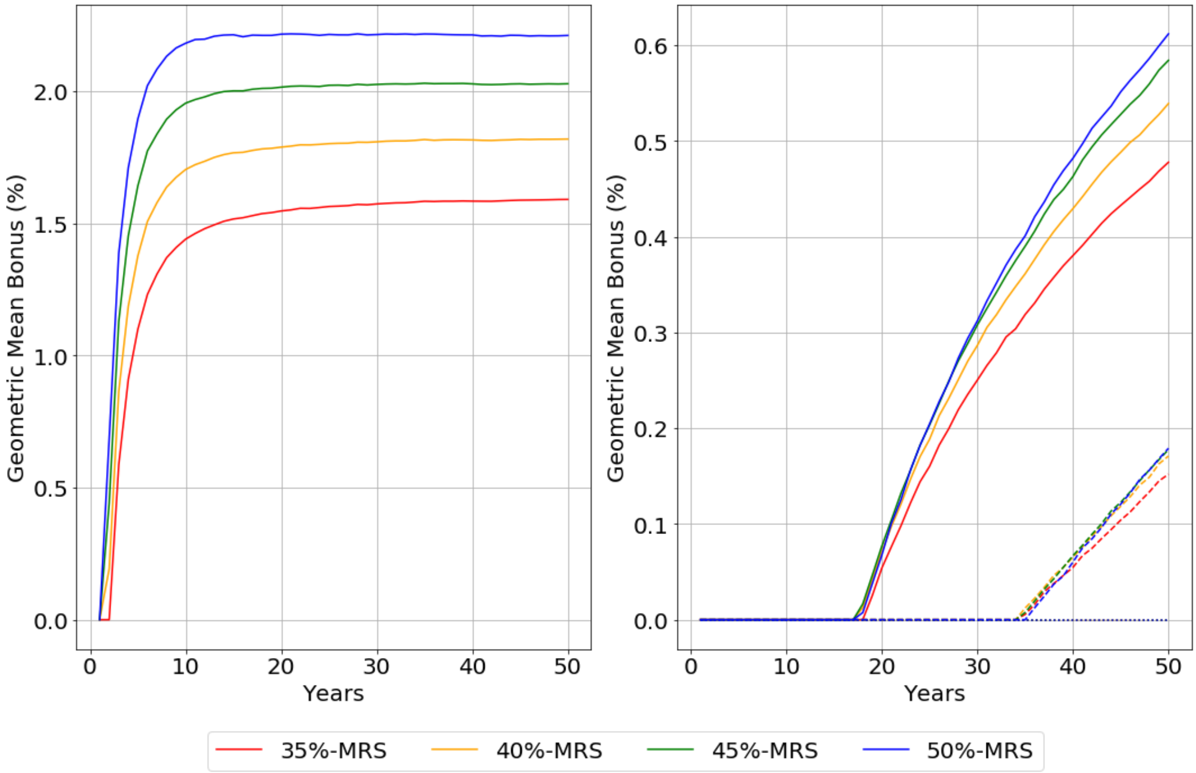

Figure 9: Median (left), and 5%, 1% and 0.1% quantiles (right) of the geometric mean bonus over a given period.

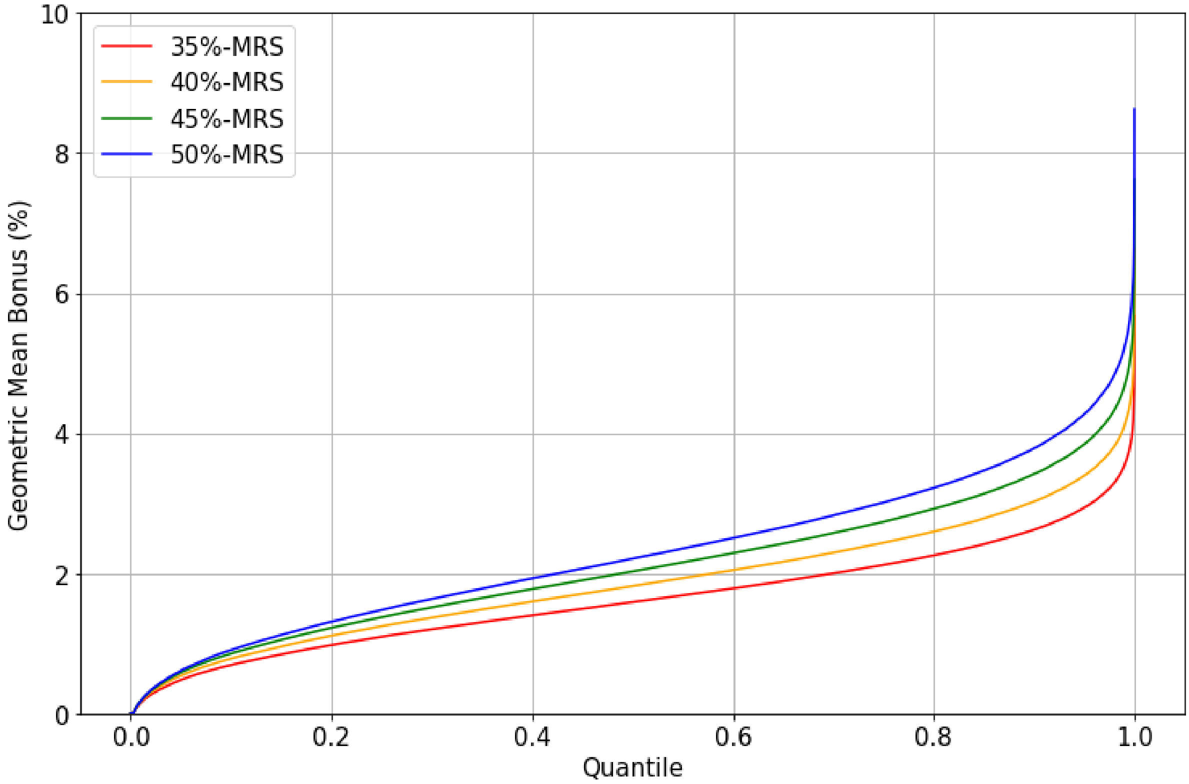

Figure 10: Simulated geometric mean bonus over the next 50 years, given ATP's pension balance mid-2019, ordered by quantiles.

However, what is not taken into account is that the zero bonuses case covers, among other scenarios, the rare scenarios where the total Surplus is lost or, less bad, the market risk budget is violated. The probabilities of such scenarios, referred to as 'bad states', are illustrated in Figure 11. Clearly, the probability of bad states increases with the risk level of the market risk strategy. Being at the lower bound for market risk, and thereby violating the market risk budget, about 11% of the time (bottom left plot) and jeopardizing the guaranteed pensions by running a negative Surplus about 1.3% of the time (bottom right plot) comes with a risk. However, choosing a lower risk strategy reduces the chance of generating 1.75% pensions bonuses and – most importantly – does not make the risk disappear. Aiming for a 1.75% geometric mean pension bonus seems within reach and balances our preferences between the short-term risks and the long-term risks appropriately. Our both quantitative and qualitative cost benefit analysis brings us to the conclusion that our 40%-MRS is reasonably prudent and holds *just* enough risk over time to be true to our investment goals.

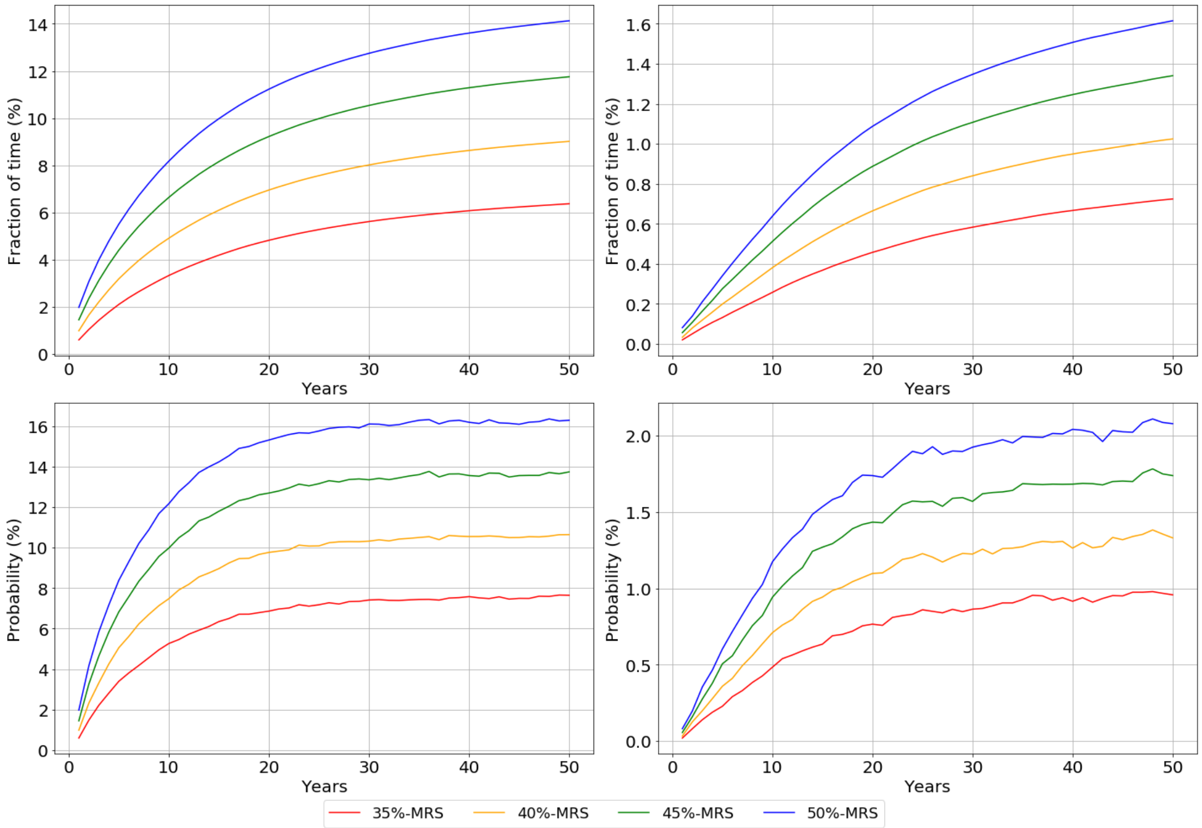

Figure 11: Top: Mean fraction of time spent violating the market risk budget (left plot) and the funding constraint (Surplus>0), respectively. Bottom: Probability of violating the market risk budget (left plot) and the funding constraint (Surplus > 0), respectively, at a given point in time.

5. Discussion

It is important to realize that even though our goal is to generate pension bonuses large enough to increase the pension benefits with the inflation over time, this is only a meaningful goal for ATP as long at the inflation is around 1.75% (or less) over the time span. The reason for this is simple: First, note that pension bonuses are generated if the Surplus (the investment portfolio) grows faster than the Reserve (the liability hedging portfolio). Second, recall that returns can be decomposed into the risk-free rate (short rate) + excess returns (risk premia). If both the Reserve and the Surplus were invested in the risk-free rate, and no additional investment risk were taken, the two portfolios would grow at the same rate. In practice, the Reserve is not invested in the risk-free rate but in rather long dated bonds and, therefore, earns excess-returns due to the term-structure premium. The Surplus, on the other hand, earns expected excess-returns from multiple risk premia, magnified by applying leverage. The term-structure premium that the Reserve earns and the multiple risk premia that the Surplus earns can logically be assumed to be *independent of the level of inflation*. Hence, the outperformance of the excess returns of the Surplus to the excess-returns of the Reserve is independent of the level of inflation and can only be increased by leveraging up the Surplus (even more) or improving the return-to-risk ratio.

On the other hand, if the required return on the investment portfolio is the level of inflation, for inflation levels above 1.75%, the corresponding required returns exceed what can be expected within our market risk budget. For higher levels, the required risk becomes so high that the median bonus starts to fall (assuming no risk limiting budget).

6. Conclusion

ATP's approach to being long-term is to combine actuarial and financial engineering with prudent management and a transparent investment structure. We protect ATP's guaranteed nominal pension benefits by splitting our investments into two portfolios: a liability hedging portfolio and a total return investment portfolio. The liability hedging portfolio generates (with a very high level of safety) the needed cashflows to pay our pension obligations. Our investment portfolio is aiming at achieving high investment returns, which translates into increased pensions over time. The investment portfolio is invested subject to a risk budget instead of subject to its cash value. Investment risk is adjusted over time to keep the probability of a negative Surplus low.

In this paper we present our dynamic management of portfolio risk which is a practical version of a CPPI strategy, incorporating both short- and long-term risk. Short-term risk is the risk of a negative surplus if a bad event happens right away. Long-term risk is the risk of not achieving our investment return goal. We demonstrate by use of our asset-liability-risk-management (ALRM) model that this way of operating balances being prudent and achieving our investment goals at the same time.

7. Appendix

In Subsection 4) of Section 4 we simulate DKK excess returns for the investment portfolio using a GARCH(1,1) model with NIG (Normal Invers Gamma) distributed innovations. Denote the relative excess returns by R_t and define the log-returns by $X_t = log(1 + R_t)$. We have

$$X_t = \mu + \sigma_t Z_t,$$

where

$$\sigma_t^2 = \omega + \alpha(X_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2,$$

and $(Z_t)_{t\in\mathbb{Z}} \sim SWN(0,1)$ is NIG distributed. The parameters are estimated using the historic DKK excess returns from our 3-factor Balanced Beta portfolio (see (Lorenzen, Gosvig, & Kronborg, 2019)). We get

$$\mu = 0.0017008238^{17}, \sigma = 0.0000142054, \alpha = 0.1428403006, \beta = 0.7548555094$$

and the values of the skew and shape parameter of the NIG distribution are

$$skew = -0.4560801968, shape = 3.5858911783.$$

All parameters are highly significant (p-values much smaller than 5%). In Figure 12 we have selected model control plots for the fit of the model. As seen from the autocorrelation plots the volatility clustering is pronounced. We also observe that the NIG distribution fits the heavy tails and skewness of the residuals (in contrast to the normal distribution). Finally, the QQ-plot looks very nice, although October-2008 is a small outlier.

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[17] We get $\mu = 0.0025130435$ before adjusting to a return-to-risk ratio of 0.38 (corresponding to a Sharpe Ratio of 0.5)

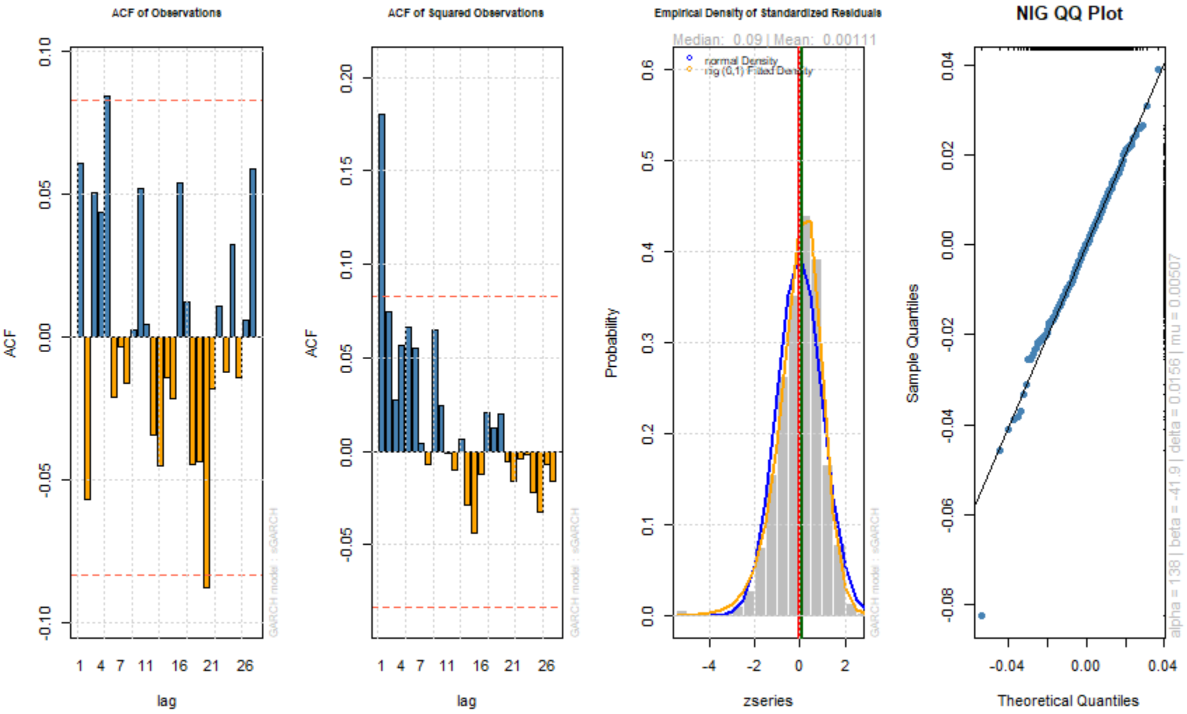

Figure 12: Model control plot for the GARCH(1,1)-NIG model fit to the historic DKK excess returns for the Balanced Beta portfolio.

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